Quantum Physics（量子物理）習題

Robert Eisberg（Second edition）

CH 01：Thermal radiation and Planck's postulate

---

1-1、At what wavelength does a cavity at $6000^0 K$ radiated most per unit wavelength?（$6000^0 K$時，一空腔輻射體的最強波長爲何？）

<解>：

---

1-2、Show that the proportionality constant in (1-4) is $\frac{4}{c}$. That is, show that the relation between spectral radiancy $R_T(\nu)$ and energy density $\rho_T(\nu)$ is $R_T(\nu)d\nu = \frac{c}{4}\rho_T(\nu)d\nu$.

<解>：(1-4) $\rho_T(\nu) \propto R_T(\nu)$

$$\rho_T(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}d\nu \;\Rightarrow\; 令\, x = \frac{h\nu}{kT}$$

$$\Rightarrow\; \rho_T(\nu)d\nu = \frac{8\pi k^4T^4}{h^3c^3}\frac{x^3}{e^x-1}dx = \frac{8\pi k^4T^4}{h^3c^3}\frac{\pi^4}{15} \quad （\text{Hint}：\int\frac{x^3dx}{e^x-1} = \frac{\pi^4}{15}）$$

$$又\;\; R_T = \sigma T^4,\;\; \sigma = \frac{2\pi^5k^4}{15c^2h^3} \;（\text{Stefan's law}）$$

$$\rho_T(\nu)d\nu = \frac{8\pi k^4T^4}{h^3c^3}\frac{\pi^4}{15} = \frac{4}{c}\left(\frac{2\pi^5k^4}{15h^3c^2}\right)T^4 = \frac{4}{c}R_T(\nu)d\nu$$

$$所以\, R_T(\nu)d\nu = \frac{c}{4}\rho_T(\nu)d\nu \;\ldots\ldots\ldots\ldots\#\#$$

---

1-3、Consider two cavities of arbitrary shape and material, each at the same temperature T, connected by a narrow tube in which can be placed color filters (assumed ideal) which will allow only radiation of a specified frequency $\nu$ to pass through. (a) Suppose at a certain frequency $\nu'$, $\rho_T(\nu')d\nu$ for cavity 1 was greater than $\rho_T(\nu')d\nu$ for cavity 2. A color filter which passes only the frequency $\nu'$ is placed in the connecting tube. Discuss what will happen in terms of energy flow. (b) What will happened to their respective temperatures? (c) Show that this would violate the second law of thermodynamic; hence prove that all blackbodies at the same temperature must emit thermal radiation with the same spectrum

東海大學物理系 量子物理習題

independent of the details of their composition.

<解>：

---

1-4、A cavity radiator at $6000^0 K$ has a hole 10.0 mm in diameter drilled in its wall. Find the power radiated through the hole in the range 5500～5510Å.（Hint : See Problem2）

（一在$6000^0 K$的輻射體有一直徑爲$10.0mm$的小洞，求由此小洞所輻射出的波長在5500～5510Å之間的功率。）

<解>：$P = A\int_{\nu_1}^{\nu_2} R_T(\nu)d\nu = \frac{1}{4}Ac\int_{\nu_1}^{\nu_2} \rho_T(\nu)d\nu = \frac{1}{4}Ac\rho_T(\nu_{av})\Delta\nu$

$$\nu_1 = \frac{c}{\lambda_1} = \frac{2.988\times10^8}{5.50\times10^{-7}} = 5.4509\times10^{14} Hz$$

$$\nu_2 = \frac{c}{\lambda_2} = \frac{2.988\times10^8}{5.51\times10^{-7}} = 5.4401\times10^{14} Hz$$

Therefore, $\nu_{av} = \frac{1}{2}(\nu_1+\nu_2) = 5.46\times10^{14} Hz$

$$\Delta\nu = \nu_2 - \nu_1 = 9.9\times10^{11} Hz$$

Since $\rho_T(\nu_{av}) = \frac{8\pi h\nu_{av}^3}{c^3}\frac{1}{e^{\frac{h\nu_{av}}{kT}}-1}$

Numerically $\frac{8\pi h\nu_{av}^3}{c^3} = 1.006\times10^{-13}$ , $\frac{h\nu_{av}}{kT} = 4.37$ , $e^{\frac{h\nu_{av}}{kT}} - 1 = 78.04$

$$\rho_T(\nu_{av}) = \frac{1.006\times10^{-13}}{78.04} = 1.289\times10^{-15}$$

The area of the hole is $A = \pi r^2 = \pi(5\times10^{-3})^2 = 7.854\times10^{-5} m^2$

Hance, finally,

$$P = \frac{1}{4}Ac\rho_T(\nu_{av})\Delta\nu = \frac{1}{4}(7.854\times10^{-5})(2.998\times10^8)(1.289\times10^{-15})(9.9\times10^{11})$$

$P = 7.51W$ ............##

---

1-5、(a) Assuming the surface temperature of the sun to be $5700^0 K$ ,use Stefan's law, (1-2), to determine the rest mass lost per second to radiation by the sun. Take the sun's diameter to be $1.4\times10^9 m$. (b) What fraction of the sun's rest mass is lost each year from electromagnetic radiation? Take the sun's rest mass to be $2.0\times10^{30} kg$ .



東海大學 Tunghai University

東海大學物理系量子物理習題

<解>：(a) $L = 4\pi R^2 \sigma T^4 = 4\pi (7\times10^8)^2 (5.67\times10^{-8})(5700)^4 = 3.685\times10^{26} W$

（$R_{sun} = 7\times10^8 m$）

$$L = \frac{d}{dt}(mc^2) = c^2\frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{L}{c^2} = \frac{3.685\times10^{26}}{(3\times10^8)^2} = 4.094\times10^9 kg/s$$

(b) The mass lost in one year is

$$\Delta M = \frac{dm}{dt}t = (4.094\times10^9)(86400\times365) = 1.292\times10^{17} kg$$

The desired fraction is , then,

$$f = \frac{\Delta M}{M} = \frac{1.292\times10^{17}}{2.0\times10^{30}} = 6.5\times10^{-14} \text{............\#\#}$$

---

1-6、In a thermonuclear explosion the temperature in the fireball is momentarily $10^7\ {}^0K$. Find the wavelength at which the radiation emitted is a maximum.
（熱核爆炸，火球溫度達到$10^7\ {}^0K$，求最大輻射強度之波長爲何？）

<解>：

---

1-7、At a given temperature, $\lambda_{\max} = 6500\,\text{Å}$ for a blackbody cavity. What will $\lambda_{\max}$ be if the temperature of the cavity wall is increased so that the rest of emission of spectral radiation is double?
（一黑體空腔在某一溫度 $\lambda_{\max} = 6500\,\text{Å}$，若溫度增加，以致使此光譜輻射能量加倍時，$\lambda_{\max}$ 應爲何？）

<解>：Stefan's law $R_T = \sigma T^4$

$$\frac{R'}{R} = 2 = \frac{\sigma T'^4}{\sigma T^4} \Rightarrow T' = \sqrt[4]{2}T$$

Wien's law :

$$\lambda'_{\max}T' = \lambda_{\max}T \Rightarrow \lambda'_{\max}(\sqrt[4]{2}T) = (6500\,\overset{0}{A})T \Rightarrow \lambda'_{\max} = \frac{6500}{\sqrt[4]{2}}\overset{0}{A} = 5466\,\overset{0}{A} \text{......\#\#}$$

---

1-8、At what wavelength does the human body emit its maximum temperature radiation? List assumptions you make in arriving at an answer.

東海大學物理學系量子物理習題



東海大學 Tunghai University

（在波長爲何時，人體所釋放出的的熱輻射爲最大？並列出你的假設。）

<解>：

---

1-9、Assuming that $\lambda_{\max}$ is in the near infrared for red heat and in the near ultraviolet for blue heat, approximately what temperature in Wien's displacement law corresponds to red heat? To blue heat?

<解>：

---

1-10、The average rate of solar radiation incident per unit area on the earth is $0.485 cal/cm^2-\min$ (or $338W/m^2$). (a) Explain the consistency of this number with the solar constant (the solar energy falling per unit time at normal incidence on a unit area) whose value is $1.94 cal/cm^2-\min$ (or $1353W/m^2$). (b) Consider the earth to be a blackbody radiating energy into space at this same rate. What surface temperature world the earth have under these circumstances?

<解>：(a) The solar constant S is defined by $S=\dfrac{L_{sun}}{4\pi r^2}$

$r=$ Earth-sun distance（地球-太陽距離）。$L_{sun}=$ rate of energy output of the sun（太陽釋放能量的速率）。令 $R=$ radius of the earth（地球半徑）。The rate P at which energy impinges on the earth is

$$P=\frac{L_{sun}}{4\pi r^2}\pi R^2=\pi R^2 S$$

The average rate, per $m^2$, of arrival of energy at the earth's surface is

$$P_{av}=\frac{P}{4\pi R^2}=\frac{\pi R^2 S}{4\pi R^2}=\frac{1}{4}S=\frac{1}{4}(1353W/m^2)=338W/m^2$$

(b) $338=\sigma T^4=(5.67\times10^{-8})T^4$

$T=277.86^0K$ …………##

<註>：課本解答 Appendix S，S-1 爲(b) $280^0K$ 。

---

1-11、Attached to the roof of a house are three solar panels, each $1m\times2m$. Assume the equivalent of 4 hrs of normally incident sunlight each day, and that all the incident light is absorbed and converted to heat. How many gallows of water can

東海大學量子物理習題

be heated from 40℃ to 120℃ each day?

<解>：

---

1-12、Show that the Rayleugh-Jeans radiation law, (1-17), is not consistent with the Wien displacement law $\nu_{\max} \propto T$, (1-3a), or $\lambda_{\max} T = const$, (1-3b).

<解>：

---

1-13、We obtain $\nu_{\max}$ in the blackbody spectrum by setting $\frac{d\rho_T(\nu)}{d\nu} = 0$ and $\lambda_{\max}$ by setting $\frac{d\rho_T(\lambda)}{d\lambda} = 0$. Why is it not possible to get from $\lambda_{\max} T = const$ to $\nu_{\max} = const \times T$ simply by using $\lambda_{\max} = \frac{c}{\nu_{\max}}$? This is, why is it wrong to assume that $\nu_{\max}\lambda_{\max} = c$, where c is the speed of light?

<解>：

---

1-14、Consider the following number : 2,3,3,4,1,2,2,1,0 representing the number of hits garnered by each member of the Baltimore Orioles in a recent outing. (a) Calculate directly the average number of hits per man. (b) Let $x$ be a variable signifying the number of hits obtained by a man, and let $f(x)$ be the number of times the number s appears. $\overline{x} = \frac{\sum_0^4 xf(x)}{\sum_0^4 f(x)}$. (c) Let $p(x)$ be the probability of the number x being attained. Show that $\overline{x}$ is given by $\overline{x} = \sum_0^4 xp(x)$

<解>：

---

1-15、Consider the function

$$f(x) = \frac{1}{10}(10-x)^2 \qquad 0 \le x \le 10$$
$$f(x) = 0 \qquad all\ other\ x$$



東海大學 Tunghai University

東海大學物理系量子物理習題

(a) From $\bar{x} = \dfrac{\int_{-\infty}^{\infty} x f(x)dx}{\int_{-\infty}^{\infty} f(x)dx}$ find the average value of x. (b) Suppose the variable x were discrete rather then continuous. Assume $\Delta x = 1$ so that x takes on only integral values 0,1,2,……,10. Compute $\bar{x}$ and compare to the result of part(a). (Hint : It may be easier to compute the appropriate sum directly rather than working with general summation formulas.) (c) Compute $\bar{x}$ for $\Delta x = 5$, i.e. $x = 0,5,10$. Compare to the result of part (a). (d) Draw analogies between the results obtained in this problem and the discussion of Section 1-4. Be sure you understand the roles played by $\bar{\varepsilon}, \Delta\varepsilon$, and $P(\varepsilon)$.

<解>：(a) $$\bar{x} = \frac{\int_0^{10} x\frac{1}{10}(10-x)^2 dx}{\int_0^{10} \frac{1}{10}(10-x)^2 dx} = \frac{\int_0^{10} x(100-20x+x^2)dx}{\int_0^{10}(100-20x+x^2)dx} = \frac{50x^2-\frac{20}{3}x^3+\frac{1}{4}x^4\Big|_0^{10}}{100x-10x^2+\frac{1}{3}x^3\Big|_0^{10}}$$

$$= \frac{50x-\frac{20}{3}x^2+\frac{1}{4}x^3\Big|_0^{10}}{100-10x+\frac{1}{3}x^2\Big|_0^{10}} = \frac{\frac{1000}{12}}{\frac{100}{3}} = 2.5$$

(b) $$\bar{x} = \frac{\sum_0^{10} x\frac{1}{10}(10-x)^2}{\sum_0^{10}\frac{1}{10}(10-x)^2} = \frac{\sum_0^{10} 100x-20x^2+x^3}{\sum_0^{10} 100-20x+x^2}$$

$$= \frac{100\times\frac{10\times11}{2}-20\times\frac{10\times11\times21}{6}+(\frac{10\times11}{2})^2}{100\times11-20\times\frac{10\times11}{2}+\frac{10\times11\times21}{6}}$$

$$= \frac{825}{385} = 2.143$$

(Hint : $\sum_1^n n = \frac{n(n+1)}{2}$, $\sum_1^n n^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_1^n n^3 = [\frac{n(n+1)}{2}]^2$)

(c) Set $x = 5n \Rightarrow$ $$\bar{x} = \frac{\sum_{n=0}^{2} 5n\frac{1}{10}(10-5n)^2}{\sum_{n=0}^{2}\frac{1}{10}(10-5n)^2} = \frac{\sum_{n=0}^{2} 20n-20n^2+5n^3}{\sum_{n=0}^{2} 4-4n+n^2}$$

東海大學物理系量子物理習題

$$= \frac{20\times\frac{2\times3}{2} - 20\times\frac{2\times3\times5}{6} + 5(\frac{2\times3}{2})^2}{4\times3-4\times\frac{2\times3}{2}+\frac{2\times3\times5}{6}} = \frac{5}{5} = 1\ldots\ldots\#\#$$

---

1-16、Using the relation $P(\varepsilon)\frac{e^{-\frac{\varepsilon}{kT}}}{kT}$ and $\int_0^\infty P(\varepsilon)d\varepsilon = 1$, evaluate the integral of (1-21) to deduce (1-22), $\bar{\varepsilon} = kT$ .

（由 $P(\varepsilon)\frac{e^{-\frac{\varepsilon}{kT}}}{kT}$ 和 $\int_0^\infty P(\varepsilon)d\varepsilon = 1$ 兩式，求 1-21 之積分，以證明 1-22 式，$\bar{\varepsilon} = kT$ ）

<解>：

---

1-17、Use the relation $R_T(\nu)d\nu = \frac{c}{4}\rho_T(\nu)d\nu$ between spectral radiancy and energy density, together with Planck's radiation law, to derive Stefan's law. That is, show that $R_T = \int_0^\infty \frac{2\pi h}{c^2}\frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}}-1} = \sigma T^4$ where $\sigma = \frac{2\pi^5 k^4}{15c^2h^3}$. (Hint : $\int_0^\infty \frac{q^3 dq}{e^q - 1} = \frac{\pi^4}{15}$)

<解>： $R_T = \int_0^\infty R_T(\nu)d\nu = \frac{2\pi h}{c^2}\int_0^\infty \frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}}-1} = \frac{2\pi k^4 T^4}{c^2h^3}\int_0^\infty \frac{xdx}{e^x - 1}$

$$= \frac{2\pi k^4 T^4}{c^2 h^3}\frac{\pi^4}{15} = \frac{2\pi^5 k^4}{15c^2h^3}T^4 = \sigma T^4$$

---

1-18、Derive the Wien displacement law, $\lambda_{\max}T = 0.2014\frac{hc}{k}$, by solving the equation $\frac{d\rho_T(\lambda)}{d\lambda} = 0$ . （Hint : Set $\frac{hc}{\lambda kT} = x$ and show that the equation quoted leads to $e^{-x} + \frac{x}{5} = 1$. Then show that $x = 4.965$ is the solution.）

<解>：

---



東海大學 Tunghai University

東海大學量子物理習題

1-19、To verify experimentally that the $3^0 K$ universal background radiation accurately fits a blackbody spectrum, it is decided to measure $R_T(\lambda)$ from a wavelength below $\lambda_{\max}$ where its value is $0.2R_T(\lambda_{\max})$ to a wavelength above $\lambda_{\max}$ where its value is again $0.2R_T(\lambda_{\max})$. Over what range of wavelength must the measurements be made?

<解>：$R_T(\lambda)=\frac{c}{4}\rho_T(\lambda)=\frac{2\pi k^5 t^5}{h^4c^3}\frac{x^5}{e^x-1}$

With $x=\frac{hc}{\lambda kT}$. At $\lambda=\lambda_{\max}$, $x=4.965$, by problem 18. Thus

$$R_T(\lambda_{\max})=42.403\pi\frac{(kt)^5}{h^4c^3}$$

Now find x such that $R_T(\lambda)=0.2R_T(\lambda_{\max})$:

$$\frac{2\pi k^5 t^5}{h^4c^3}\frac{x^5}{e^x-1}=(0.2)42.403\pi\frac{(kt)^5}{h^4c^3}$$

$$\frac{x^5}{e^x-1}=4.2403$$

$x_1=1.882$，$x_2=10.136$

Numerically, $\lambda=\frac{hc}{kT}\frac{1}{x}=\frac{(6.626\times10^{-34})(2.998\times10^8)}{(1.38\times10^{-23})(3)}\frac{1}{x}$

$$\lambda=\frac{4.798\times10^{-3}}{x}$$

So that $\lambda_1=\frac{4.798\times10^{-3}}{1.882}=2.55mm$

$$\lambda_2=\frac{4.798\times10^{-3}}{10.136}=0.473mm\ldots\ldots\#\#$$

---

1-20、Show that, at he wavelength $\lambda_{\max}$, where $\rho_T(\lambda)$ has its maximum $\rho_T(\lambda_{\max})=170\pi\frac{(kT)^5}{(hc)^4}$.

<解>：If $x=\frac{hc}{\lambda_{\max}kT}$, then, by Problem 18, $e^{-x}+\frac{x}{5}=1 \Rightarrow e^x-1=\frac{x}{5-x}$

Hence, $\rho_T(\lambda_{\max})=\frac{8\pi kT}{\lambda_{\max}^4}(5-x)$.

But, $x=4.965$, $\frac{1}{\lambda_{\max}^4}=(4.965\frac{kT}{hc})^4$

東海大學物理學系量子物理習題

Upon substitution, there give $\rho_T(\lambda_{\max}) = 170\pi \frac{(kT)^5}{(hc)^4}$ ……##

---

1-21、Use the result of the preceding problem to find the two wavelengths at which $\rho_T(\lambda)$ has a value one-half the value at $\lambda_{\max}$. Give answers in terms of $\lambda_{\max}$.

<解>：By Problem 20, $\rho_T(\lambda_{\max}) = 170\pi \frac{(kT)^5}{(hc)^4}$.

So that the wavelengths sought must satisfy $\frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} = \frac{1}{2} \cdot 170\pi \frac{(kT)^5}{(hc)^4}$.

Again let $x = \frac{hc}{\lambda kT}$.

In terms of x, the preceding equation becomes $\frac{x^5}{e^x - 1} = \frac{170}{16}$.

Solutions are $x_1 = 2.736$ ; $x_2 = 8.090$.

Since, for $\lambda_{\max}$, $x = 4.965$, these solutions give $\lambda_1 = 1.815\lambda_{\max}$, $\lambda_2 = 0.614\lambda_{\max}$ ……##

---

1-22、A tungsten sphere 2.30cm in diameter is heated to 2000℃. At this temperature tungsten radiates only about 30% of the energy radiated by a blackbody of the same size and temperature. (a) Calculate the temperature of a perfectly black spherical body of the same size that radiates at the same rate as the tungsten sphere. (b) Calculate the diameter of a perfectly black spherical body at the same temperature as the tungsten sphere that radiates at the same rate.

<解>：(a) $30\% \times \sigma T_{tungsten}^4 = \sigma T_{black}^4 \Rightarrow 30\% \times (2000 + 273) = T^4$

$T = 1682\,^0K = 1409\,^0C$

(b) $30\% \times \sigma T_{tungsten}^4 \times 4\pi(2.3cm)^2 = \sigma T_{black}^4 \times 4\pi r^2$

$T_{tungsten} = T_{black} \Rightarrow r = 1.26cm$ ……##

---

1-23、(a) Show that about 25% of the radiant energy in a cavity is contained within



東海大學 Tunghai University

東海大學物理系 量子物理習題

wavelengths zero and $\lambda_{max}$; i.e., show that $\dfrac{\int_0^{\lambda_{max}} \rho_T(\lambda)d\lambda}{\int_0^{\infty} \rho_T(\lambda)d\lambda} \simeq \dfrac{1}{4}$ (Hint : $\dfrac{hc}{\lambda_{max}kT} = 4.965$; hence Wien's approximation is fairly accurate in evaluating the integral in the numerator above.) (b) By the percent does Wien's approximation used over the entire wavelength range overestimate or underestimate the integrated energydensity?

<解>：

---

1-24、Find the temperature of a cavity having a radiant energy density at 2000Å that is 3.82 times the energy density at 4000Å.

<解>：Let $\lambda' = 200nm$, $\lambda'' = 400nm$; then

$$\frac{1}{\lambda'^5}\frac{1}{e^{\frac{hc}{\lambda' kT}}-1} = 3.82\times\frac{1}{\lambda''^5}\frac{1}{e^{\frac{hc}{\lambda'' kT}}-1} \Rightarrow \frac{e^{\frac{hc}{\lambda'' kT}}-1}{e^{\frac{hc}{\lambda' kT}}-1} = 3.82\times(\frac{\lambda'}{\lambda''})^5$$

Numerically, $\dfrac{hc}{\lambda' k} = \dfrac{(6.626\times10^{-34})(2.988\times10^{8})}{(2\times10^{-7})(1.38\times10^{-23})} = 71734K$

$$\frac{hc}{\lambda'' k} = \frac{(6.626\times10^{-34})(2.988\times10^{8})}{(4\times10^{-7})(1.38\times10^{-23})} = 35867K$$

so that $\dfrac{e^{\frac{35867}{T}}-1}{e^{\frac{71734}{T}}-1} = 3.82\times(\dfrac{1}{2})^5 = 0.1194$

Let $x = e^{\frac{35867}{T}}$; then $\dfrac{x-1}{x^2-1} = 0.1194 = \dfrac{1}{x+1} \Rightarrow x = 7.375 = e^{\frac{35867}{T}}$

$$T = \frac{35867}{\ln 7.375} = 17950K \;\ldots\ldots\ldots\ldots\#\#$$

<註>：課本解答 Appendix S，S-1 爲 $18020^0K$ 。

---

東海大學量子物理習題

其他補充題目

1-001、(a) 簡單敘述黑體輻射的性質，並說明空槍的哪一部份可代表黑體。(b) 黑體輻射的輻射強度爲 $R(\lambda)=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$，試討論 $\lambda\to 0$，$\lambda\to\infty$ 的情況下的簡化式，並做圖說明之。

<解>：(a) Blackbody（黑體），一完全輻射體或一完全的吸收體，因其不反射任何光譜，使得外表爲黑色，故稱其爲黑體。一般常以空腔輻射實驗來做黑體頻譜分析及輻射能量分佈的研究。
空腔表面的小孔可視爲一黑體，因爲可視它爲一完全的輻射體。

(b) 黑體輻射的輻射強度爲 $R(\lambda)=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$

$$R(\lambda\to 0)=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\Big|_{\lambda\to 0}=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})e^{-\frac{hc}{\lambda kT}}\ \ldots\text{（Wien's law）}$$

$$R(\lambda\to\infty)=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\Big|_{\lambda\to\infty}$$

$$=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{1+\frac{hc}{\lambda kT}-1}$$

$$=\frac{c}{4}(\frac{8\pi kT}{\lambda^4})\ \ldots\text{（Rayleigh-Jeans law）}\ \ldots\ldots\#\#$$

1-002、Use Planck's radiation theory show (a) Wien displacement law : $\lambda_m T=2.898\times10^{-3}mK$ . (b) Stefan's law : $R=\sigma T^4$, $\sigma=\frac{2\pi^5k^4}{15c^2h^3}$

<解>：(a) 普朗克黑體輻射公式：$\rho(\lambda)d\lambda=\frac{8\pi}{\lambda^5}\frac{hc}{e^{\frac{hc}{\lambda kT}}-1}d\lambda$

由微積分求極值法，可知道 $\frac{d\rho(\lambda)}{d\lambda}\Big|_{\lambda_m}=0\ \ldots\ldots(1)$

東海大學物理系量子物理習題

所以 $\dfrac{d\rho(\lambda)}{d\lambda}=\dfrac{d}{d\lambda}[\dfrac{8\pi}{\lambda^5}\dfrac{hc}{e^{\frac{hc}{\lambda kT}}-1}]=8\pi hc\dfrac{d}{d\lambda}[\dfrac{\lambda^{-5}}{e^{\frac{hc}{\lambda kT}}-1}]$

$$=8\pi hc[\frac{-5(e^{\frac{hc}{\lambda kT}}-1)\lambda^{-6}+\lambda^{-5}\frac{hc}{\lambda kT}e^{\frac{hc}{\lambda kT}}}{(e^{\frac{hc}{\lambda kT}}-1)^2}]$$ ……代入(1)式

得 $[-5(e^{\frac{hc}{\lambda kT}}-1)+\dfrac{hc}{\lambda kT}e^{\frac{hc}{\lambda kT}}]\Big|_{\lambda_m}=0$ ……(2)

令 $x=\dfrac{hc}{\lambda kT}$，則(2)式可寫成 $-5(e^x-1)+xe^x=0$

令 $\begin{cases} y=-5(e^x-1) \\ y=xe^x \end{cases}$ 畫圖求解

得到 Wien displacement law $\lambda_m T=\dfrac{hc}{xk}=2.898\times10^{-3}mK$

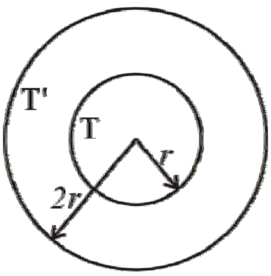
(b) 輻射強度： $R(\lambda)=\dfrac{c}{4}\rho(\lambda)=\dfrac{c}{4}(\dfrac{8\pi hc}{\lambda^5})\dfrac{1}{e^{\frac{hc}{\lambda kT}}-1}$

總輻射強度： $R_T=\displaystyle\int_0^\infty R(\lambda)d\lambda=\dfrac{8\pi hc^2}{4}\int_0^\infty\dfrac{d\lambda}{\lambda^5(e^{\frac{hc}{\lambda kT}}-1)}$

令 $x=\dfrac{hc}{\lambda kT}$，得到 $R_T=\sigma T^4$， $\sigma=\dfrac{2\pi^5k^4}{15c^2h^3}$ （Stefan's law） ……##

---

1-003、A sphere of radius $r$ is maintained at a surface temperature T by an internal heat source. The sphere is surrounded by a thin concentric shell of radius $2r$. Both object emit and absorb as blackbodies. What is the temperature of the shell?




<解>： $R_T=\sigma T^4$ （Stefan's law）

對於小球而言，其爲熱源，可視爲一完全輻射體，其總功率即爲大球吸收

東海大學物理學系量子物理習題

之總功率

$P_a = 4\pi r^2 R = 4\sigma\pi r^2 T^4$ （單面吸收）

對於大球而言，既吸收又輻射，輻射總功率為：

$P_e = 2[4\pi(2r)^2]R = 32\sigma\pi r^2 T'^4$ （雙面輻射）

因大球為一黑體，故在平衡時，$P_a = P_e$

所以外球殼的溫度為：$T' = \sqrt[4]{\frac{T^4}{8}} = 0.595T$ ……##

---

1-004、Assuming that the probability distribution for occupation of the oscillator is given by Boltzmann law. (a) Show that the mean energy E of an oscillator for a given temperature is $\frac{h\nu}{e^{\frac{hc}{\lambda kT}}-1}$. (b) Show that for a sufficiently high temperature this expression becomes equal to the classical one. Give the other of magnitude of this temperature.

<解>：(a) $\bar{\varepsilon} = \frac{\sum \varepsilon P(\varepsilon)}{\sum P(\varepsilon)} = \frac{\sum (nh\nu)\frac{1}{kT}e^{-\frac{nh\nu}{kT}}}{\sum \frac{1}{kT}e^{-\frac{nh\nu}{kT}}} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

(b) 若溫度 $T > \frac{h\nu}{k} = \frac{hc}{\lambda k}$

則上式 $\bar{\varepsilon} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1} \cong \frac{h\nu}{1+\frac{h\nu}{kT}-1} = kT$ ……回歸古典極限結果。……##

---

1-005、Cosmic background radiation peaks at a wavelength of about 1 mm. What is the temperature of the universe? (Hint: use Wien's law)

<解>：According to the Wien's law

$\lambda_{\max} T = 0.2014\frac{hc}{k_B} = 2.898\times10^{-3} m\,^0K$

For $\lambda_{\max} = 1mm = 10^{-3} m$

$T = 2.898\,^0K \approx 3\,^0K$ ……##

東海大學量子物理習題

1-006、What is the wavelength of a photon whose energy is equal to the rest mass energy of an electron?

<解>： $m_0c^2 = E = h\nu = h\frac{c}{\lambda}$

$$\lambda = \frac{hc}{m_0c^2} = 0.0243\overset{0}{A}\ldots\ldots\#\#$$

東海大學物理學系量子物理習題