Quantum Physics（量子物理）習題

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CH 02：Photons-particlelike properties of radiation

2-1、(a) The energy required to remove an electron from sodium is 2.3eV. Does sodium show a photoelectric effect for yellow light, with $\lambda = 5890$ Å? (b) What is the cutoff wavelength for photoelectric emission from sodium?

<解>：

2-2、Light of a wavelength 2000Å falls on an aluminum surface. In aluminum 4.2eV are required to remove an electron. What is the kinetic energy of (a) the fastest and (b) the slowest emitted photoelectrons? (c) What is the stopping potential? (d) What is the cutoff wavelength for aluminum? (e) If the intensity of the incident light is $2.0W/m^2$, what is the average number of photons per unit time per unit area that strike the surface?

<解>：(a) 2.0eV

(b) zero

(c) 2.0V

(d) 2950Å

(e) $2.0\times10^{14}/cm^2-\sec$

2-3、The work function for a clean lithium surface is 2.3eV. Make a rough plot of the stopping potential V_0 versus the frequency of the incident light for such a surface, indicating its important features.

<解>：

2-4、The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength $\lambda = 4910$ Å is 0.71V. When the incident wavelength is changed the stopping potential is found to be 1.43V. What is the new wavelength?

<解>：3820Å

2-5、In a photoelectric experiment in which monochromatic light and sodium photocathode are used, we find a stopping potential of $1.85V$ for $\lambda = 3000$ Å and of $0.82V$ for $\lambda = 4000$ Å. From these data determine (a) a value for Planck's constant, (b) the work function of sodium in electron volts, and (c) the threshold wavelength for sodium.

<解>：The photoelectric equation is $hc = eV_0\lambda + w_0\lambda$

with $V_0 = 1.85V$ for $\lambda = 300nm$, and $V_0 = 0.82V$ for $\lambda = 400nm$

$$hc = 8.891\times10^{-26} + 3\times10^{-7}w_0$$

$$hc = 5.255\times10^{-26} + 4\times10^{-7}w_0$$

Hence, $8.891\times10^{-26} + 3\times10^{-7}w_0 = 5.255\times10^{-26} + 4\times10^{-7}w_0$

$$w_0 = 3.636\times10^{-19}J = 2.27eV$$

(b) Therefore, $hc = 8.891\times10^{-26} + (3\times10^{-7})\times(3.636\times10^{-19}) = 19.799\times10^{-26}J-m$

(a) $h = \dfrac{19.799\times10^{-26}}{2.988\times10^{8}} = 6.604\times10^{-34}J-s$

(c) $w_0 = \dfrac{hc}{\lambda_0}$, $3.636\times10^{-19} = \dfrac{19.799\times10^{-26}}{\lambda_0}$

$\lambda_0 = 5.445\times10^{-7}m = 544.5nm$ …………##

2-6、Consider light shining on a photographic plate. The light will be recorded if it dissociates an AgBr molecule in the plate. The minimum energy to dissociate this molecule is of the order of 10^{-19} joule. Evaluate the cutoff wavelength greater than which light will not be recorded.

<解>：

2-7、The relativistic expression for kinetic energy should be used for the electron in the photoelectric effect when $\dfrac{v}{c} > 0.1$, if errors greater than about 1% are to be avoied. For photoelectrons ejected from an aluminum surface ($\omega_0 = 4.2eV$) what is the amallest wavelength of an incident photo for which the classical expression may be used?

<解>：

2-8、X rays with $\lambda = 0.71\,\text{Å}$ eject photoelectrons from a gold foil. The electrons from circular paths of radius r in a region of magnetic induction B. Experiment shows that $rB = 1.88\times10^{-4}\,tesla-m$. Find (a) the maximum kinetic energy of the photoelectrons and (b) the work done in removing the electron from the gold foil.

<解>：In a magnetic field $r = \dfrac{mv}{eB}$

$$p = mv = erB = (1.602\times10^{-19})(1.88\times10^{-4}) = 3.012\times10^{-23}\,kg-m/s$$

$$p = \frac{(3.012\times10^{-23})(2.988\times10^{8})}{c(1.602\times10^{-13})} = \frac{0.05637\,MeV}{c}$$

Also, $E^2 = p^2c^2 + E_0^2$

$$E^2 = (0.05637)^2 + (0.511)^2$$

$$E = 0.5141\,MeV$$

Hence, (a) $K = E - E_0 = 0.5141 - 0.5110 = 0.0031\,MeV = 3.1\,keV$

(b) The photon energy is

$$E_{ph}(eV) = \frac{1240}{\lambda(nm)} = \frac{1240}{0.071} = 0.0175\,MeV$$

$$w_0 = E_{ph} - K = 17.5 - 3.1 = 14.4\,keV \;\ldots\ldots\#\#$$

2-9、(a) Show that a free electron cannot absorb a photon and conserve both energy and momentum in the process. Hence, the photoelectric process requires a bund electron. (b) In the Compton effect, the electron can be free . Explain.

<解>：(a) Assuming the process can operate, apply conservation of mass-energy and of momentum： $h\nu + E_0 = K + E_0 \rightarrow h\nu = K$

$$\frac{h\nu}{c} + 0 = p$$

These equations taken together imply that $p = \dfrac{K}{c}$ ……(1)

But, for an electron, $E^2 = p^2c^2 + E_0^2$

$$(K+E_0)^2 = p^2c^2 + E_0^2 \Rightarrow p = \frac{\sqrt{K^2+2E_0K}}{c} \text{......(2)}$$

(1) and (2) can be satisfied together only if $E_0 \neq 0$,which is not true for an electron.

(b) In the Compton effect, a photon is present after the collision; this allows the conservation laws to hold without contradiction.

2-10、Under ideal conditions the normal human eye will record a visual sensation at 5500Å if as few as 100 photons are absorbed per seconds. What power level does this correspond to?

<解>：$3.6\times10^{-17}W$

2-11、An ultraviolet lightbulb, emitting at 4000Å, and an infrared lightbulb, emitting at 7000Å, each are rated at 40W. (a) Which bulb radiates photons at the greater rate, and (b) how many more photons does it produce each second over the other bulb?

<解>：

2-12、Solar radiation falls on the earth at a rate of $1.94 cal/cm^2-\min$ on a surface normal to the incoming rays. Assuming an average wavelength of 5500Å, how many photons per $cm^2-\min$ is this?

<解>：$1.235\times10^{20}Hz$,
2.427×10^{-2} Å,
$2.731\times10^{-22}kg-m/\sec$

2-13、What are the frequency, wavelength, and momentum of a photon whose energy equals the rest mass energy of an electron?

<解>：

2-14、In the photon picture of radiation, show that if beams of radiation of two different wavelengths are to have the same intensity (or energy density) then the numbers of the photons per unit cross-sectional areas per sec in the beams are in the same ratio as the wavelength.

<解>：Let n = number of photons per unit volume. In time t, all photons initially a distance <ct will cross area A normal to the beam direction. Thus $I=\frac{Energy}{At}=\frac{n(h\nu)A(ct)}{At}=nhc\nu=\frac{nhc^2}{\lambda}$. For two beams of wavelengths λ_1 and λ_2 with $I_1=I_2$, $\frac{I_1}{I_2}=1=\frac{\frac{n_1}{\lambda_1}}{\frac{n_2}{\lambda_2}}\Rightarrow\frac{n_1}{n_2}=\frac{\lambda_1}{\lambda_2}$, and therefore $\frac{\frac{n_1}{At}}{\frac{n_2}{At}}=\frac{\lambda_1}{\lambda_2}$. The energy density is $\rho=nh\nu=\frac{nhc}{\lambda}$. Since this differs from I only by the factor c (which is the same for both beams), then if $\rho_1=\rho_2$, the equation above holes again.

2-15、Derive the relation $\cot\frac{\theta}{2}=(1+\frac{h\nu}{m_0c^2})\tan\varphi$ between the direction of motion of the scattered photon and recoil electron in the Compton effect.

<解>：

2-16、Derive a relation between the kinetic energy K of the recoil electron and the energy E of the ncident photon in the Compton effect. One from of the relation is $\frac{K}{E}=\frac{(\frac{2h\nu}{m_0c^2})\sin^2\frac{\theta}{2}}{1+(\frac{2h\nu}{m_0c^2})\sin^2\frac{\theta}{2}}$. （Hint : See Example 2-4.）

<解>：

2-17、Photons of wavelength 0.024Å are incident on free electrons. (a) Find the wavelength of a photon which is scattered 30^0 from the incident direction and the kinetic energy imparted to the recoil electron. (b) Do the same if the scattering angle is 120^0. （Hint : See Example 2-4.）

<解>：

2-18、An x-ray photon of initial energy $1.0\times10^5 eV$ traveling in the $+x$ direction is incident on a free electron at rest. The photon is scattered at right angles into the $+y$ direction.Find the components of momentum of the recoiling electron.

<解>：

2-19、(a) Show that $\frac{\Delta E}{E}$, the fractional change in photo energy in the Compton effect,equals $\frac{h\nu'}{m_0c^2}(1-\cos\theta)$. (b) Plot $\frac{\Delta E}{E}$ versus θ and interpret the curve physically.

<解>：

2-20、What fractional increase in wavelength leads to a 75% loss of photon energy in a Compton collision?

<解>：300%

2-21、Througth what angle must a 0.20MeV photon be scattered by a free electron so that it loses 10% of its energy?

<解>：44^0

2-22、What is the maximum possible kinetic energy of a recoiling Compton electron in terms of the incident photon energy $h\nu$ and the electron's rest energy m_0c^2?

<解>：

2-23、Determine the maximum wavelength shift in the Compton scattering of photons from protons.

<解>：2.64×10^{-5} Å

2-24、(a) Show that the short wavelength cutoff in the x-ray continuous spectrum is given by $\lambda_{\min} = 12.4$ Å/V, where V is applied voltage in kilovolts. (b) If the voltage across an x-ray tube is 186kV what is $\lambda_{\min}$?

<解>：

2-25、(a) What is the minimum voltage across x-ray tube that will produce an x ray having the Compton wavelength? A wavelength of 1Å? (b) What is the minimum voltage needed across an x-ray tube if the subsequent bremsstrahlung radiation is to be capable of pair production?

<解>：

2-26、A 20KeV electron emits two bremsstrahlung photons as it is being brought to rest in two successive deceleration. The wavelength of the second photon is 1.30Å longer than the wavelength of the first. (a) What was the energy of the electron after the first deceleration, (b) what are the wavelength of the photons?

<解>：Set $K_i = 20keV$, $K_f = 0$; $K_1 =$ electron kinetic energy after the first deceleration; then $\dfrac{hc}{\lambda_1} = K_i - K_1$

$$\frac{hc}{\lambda_2} = K_1 - K_f = K_1$$

$$\lambda_2 = \lambda_1 + \Delta\lambda$$

with $\Delta\lambda = 0.13nm$ since $hc = 1.2400keV - nm$

$$\Rightarrow \frac{1.2400}{\lambda_1} = 20 - K_1$$

$$\frac{1.2400}{\lambda_2} = K_1$$

$$\lambda_2 = \lambda_1 + 0.13$$

solving yields, (a) $K_1 = 5.720keV$

(b) $\lambda_1 = 0.0868nm = 0.868\overset{0}{A}$

$\lambda_2 = 0.2168nm = 2.168\overset{0}{A}$ ……##

<註>：課本解答 Appendix S，S-1 爲 (26a)5.725keV (26b)0.870 Å, 2.170 Å

2-27、A γ ray creates an electron-positron pair. Show directly that, without the presence of a third body to take up some of the momentum, energy and momentum cannot both be conserved. （Hint : Set the energies equal and show that leads to unequal momenta beford and after the interaction.）

<解>：

2-28、A γ ray can produce an electron-positron pair in the neighborhood of an electron at rest as well as a nucleus. Show that in this case the threshold energy is $4m_0c^2$.

（Hint : Do not ignore the recoil of the original electron, but assume that all three particles move off together.）

<解>：

2-29、A particular pair is produced such that the positron is at rest and the electron has a kinetic energy of 1.0MeV moving in the direction of flight of the pair-producting photon. (a) Neglecting the energy transferred to the nucleus of the nearby atom, find the energy of the incident photon. (b) What percentage of the photon's momentum is transferred to the nucleus?

<解>：(a) $E + M_0c^2 = M_0c^2 + 2m_0c^2 + K$

$E = 2\times 0.511 + 1 = 2.022MeV$

(b) $p = \frac{E}{c} = \frac{2.022MeV}{c}$

$P_+ = 0$; $P_- = \frac{1}{c}(K^2 + 2m_0c^2K)^{1/2} = \frac{1}{c}(1^2 + 2\times 0.511\times 1)^{1/2} = \frac{1.422MeV}{c}$

$P = 2.022 - 1.422 = \frac{0.600MeV}{c}$

Transferred 百分比 $= \frac{0.600}{2.022}\times 100\% = 29.7\%$ ....##

2-30、Assume that an electron-positron pair is formed by a photon having the threshold energy for the process. (a) Calculate the momentum transferred to the nucleus in the process. (b) Assume the nucleus to be that of a lead atom and compute the kinetic energy of the recoil nucleus. Are we justified in neglecting this energy compared to the threshold energy assumed above?

<解>：(a) $5.46\times 10^{-22}\,kg-m/\sec$

(b) 2.71eV, yes

2-31、An electron-position pair at rest annihilabe, creating two photos. At what speed must an observer move along the line of the photons in order that the wavelength of one photon be twice that of the other?

<解>：Use the Doppler shift to convert the given wavelengths to wavelengths as seen in the rest frame of the pair :

$2\lambda_1' = \lambda_2'$

$2\lambda(\frac{c-v}{c+v})^{1/2} = \lambda(\frac{c+v}{c-v})^{1/2}$

set $\beta \equiv \frac{v}{c} \Rightarrow 2\lambda(\frac{1-\beta}{1+\beta})^{1/2} = \lambda(\frac{1+\beta}{1-\beta})^{1/2} \Rightarrow 4\times\frac{1-\beta}{1+\beta} = \frac{1+\beta}{1-\beta}$

$3\beta^2 - 10\beta + 3 = 0$

$\beta = \frac{1}{3},\ v = \frac{c}{3}$......##

2-32、Show that the results of Example 2-8, expressed in terms of ρ and t, are valid independent of the assumed areas of the slab.

<解>：

2-33、Show that the attenuation length Λ is just equal to the average distance a photon will travel before being scattered or absorbed.

<解>：The number of particles stopped/scattered between distances x and $x+dx$ is $dI(x)=\sigma I(x)$. Hence, for a very thick slab that ultimately stops/scatters all the incident particles, the average distance a particle travels is

$$x_{av}=\frac{\int xdI}{\int dI}=\frac{\sigma\rho\int xIdx}{\sigma\rho\int Idx}=\frac{\int xe^{-\sigma\rho x}dx}{\int e^{-\sigma\rho x}dx}=\frac{1}{\sigma\rho}=\Lambda$$, the limits on all x integrals being $x=0$ to $x=\infty$.

2-34、Use the data of Figure 2-17 to calculate the thickness of a lead slab which will attenuate a beam of 10keV x rays by a factor 100.

<解>：