Quantum Physics（量子物理）習題

Robert Eisberg（Second edition）

CH 03：De Broglie's postulate-wavelike properties of particles

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3-1、A bullet of mass 40g travels at 1000m/sec. (a) What wavelength can we associate with it? (b) Why does the wave nature of the bullet not reveal itself through diffraction effects?

<解>：

3-2、The wavelength of the yellow spectral emission of sodium is 5890Å. At what kinetic energy would an electron have the same de Broglie wavelength?

<解>：$4.34\times10^{-6}\,eV$

3-3、An electron and a photon each have a wavelength of 2.0Å. What are their (a) momenta and (b) total energies? (c) Compare the kinetic energies of the electron and the photon.

<解>：

3-4、A nonrelativistic particle is moving three times as fast as an electron. The ratio of their de Broglie wavelength, particle to electron, is 1.813×10^{-4}. Identify the particle.

<解>：$$\frac{\lambda_x}{\lambda_e}=\frac{\dfrac{h}{m_x v_x}}{\dfrac{h}{m_e v_e}}=\frac{m_e}{m_x}\frac{v_e}{v_x}\;\Rightarrow\;1.813\times10^{-4}=\frac{9.109\times10^{-31}\,kg}{m_x}\left(\frac{1}{3}\right)$$

$$\Rightarrow\; m_x=1.675\times10^{-27}\,kg$$

Evidently, the particle is a neutron……##

3-5、A thermal neutron has a kinetic energy $\frac{3}{2}kT$ where T is room temperature, 300^0K. Such neutrons are in thermal equilibrium with normal surroundings. (a) What is the energy in electron volts of a thermal neutron? (b) What is its de Broglie wavelength?

<解>：

3-6、A particle moving with kinetic energy equal to its rest energy has a de Broglie wavelength of 1.7898×10^{-6} Å. If the kinetic energy doubles, what is the new de Broglie wavelength?

<解>：1.096×10^{-6} Å

3-7、(a) Show that the de Broglie wavelength of a particle, of charge e, rest mass m_0, moving at relativistic speeds is given as a function of the accelerating potential V as $\lambda=\frac{h}{\sqrt{2m_0eV}}(1+\frac{eV}{2m_0c^2})^{-1/2}$ (b) Show how this agrees with $\lambda=\frac{h}{p}$ in the nonrelativistic limit.

<解>：(a) $E^2=p^2c^2+E_0^2$; $(K+E_0)^2=p^2c^2+E_0^2$,

$$p=\frac{1}{c}(K^2+2KE_0)^{1/2}=\frac{\sqrt{2KE_0}}{c}(1+\frac{K}{2E_0})^{1/2}$$

But $K=eV$ and $E_0=m_0c^2$, so that

$$\frac{\sqrt{2KE_0}}{c}=(\frac{2KE_0}{c^2})^{1/2}=(\frac{2(eV)(m_0c^2)}{c^2})^{1/2}=\sqrt{2m_0eV}$$

And $\frac{K}{2E_0}=\frac{eV}{2m_0c^2}$. Therefore, $\lambda=\frac{h}{p}=\frac{h}{\sqrt{2m_0eV}}(1+\frac{eV}{2m_0c^2})^{-1/2}$ ……##

(b) Nonrelativistic limit： $eV<<m_0c^2$; set $1+\frac{eV}{2m_0c^2}=1$ to get

$$\lambda=\frac{h}{(2m_0eV)^{1/2}}=\frac{h}{(2m_0K)^{1/2}}=\frac{h}{m_0v}$$ ……##

3-8、Show that for a relativistic particle of rest energy E_0, the de Broglie wavelength in Å is given by $\lambda = \dfrac{1.24\times10^{-2}}{E_0(MeV)}\dfrac{(1-\beta^2)^{1/2}}{\beta}$ where $\beta = \dfrac{v}{c}$.

<解>：$$\lambda = \frac{h}{mv} = \frac{h\sqrt{1-\frac{v^2}{c^2}}}{m_0 v} = \frac{hc\sqrt{1-\frac{v^2}{c^2}}}{(m_0c^2)(\frac{v}{c})} = \frac{hc}{E_0}\frac{\sqrt{1-\beta^2}}{\beta}$$

Numerically

$$hc = \frac{(6.626\times10^{-34}J-s)(2.998\times10^{8}m/s)}{(1.602\times10^{-13}J/MeV)(10^{-9}m/nm)} = 1.2358\times10^{-3}MeV-nm \approx 1.2400\times10^{-3}MeV-nm$$

$$\lambda(nm) = \frac{1.2400\times10^{-3}MeV-nm}{E_0(MeV)}\frac{(1-\beta^2)^{1/2}}{\beta}$$

$$\therefore\ \lambda(\overset{0}{A}) = \frac{1.24\times10^{-2}}{E_0(MeV)}\frac{(1-\beta^2)^{1/2}}{\beta}(\overset{0}{A})\ \ldots\ldots\#\#$$

3-9、Determine at what energy, in electron volts, the nonrelativistic expression for the de Broglie wavelength will be in error by 1% for (a) an electron and (b) a neutron. (Hint : See Problem 7.)

<解>：

3-10、(a) Show that for a nonrelativistic particle, asmall charge in speed leads to a change in de Broglie wavelength given from $\dfrac{\Delta\lambda}{\lambda_0} = \dfrac{\Delta v}{v_0}$. (b) Derive an analogous formula for a relativistic particle.

<解>：

3-11、The 50-GeV (i.e., $50\times10^{9}eV$) electron accelerator at Stanford University provides an electron beam of very short wavelength, suitable for probing the details of nuclear structure by scattering experiments. What is this wavelength and how does it compare to the siza of an average nucleus? (Hint : At these energies it is simpler to use the extreme relativistic relationship between momentum and energy, namely $p=\frac{E}{c}$. This is the same relationship used for photons, and it is justified whenever the kinetic energy of a particle is very much greater than its rest energy m_0c^2, as in this case.)

<解>：

3-12、Make a plot of de Broglie wavelength against kinetic energy for (a) electrons and (b) protons. Restrict the range of energy values to those in which classical mechanics appliesreasonably well. A convenient criterion is that the maximum kinetic energy on each plot be only about, say, 5% of the rest energy m_0c^2 for the particular particle.

<解>：

3-13、In the experiment of Davission and Germer, (a) show that the second- and third-order diffracted beams, corresponding to the strong first maximum of Figure 3-2, cannot occur and (b) find the angle at which the first-order diffracted beam would occur if the accelerating potential were changed from 54 to 60V? (c) What accelerating potential is needed to produce a second-order diffracted beam at 50^0?

<解>：

3-14、Consider a crystal with the atoms arranged in a cubic array, each atom a distance 0.91Å from its nearest neighbor. Examine the conditions for Bragg reflection from atomic planes connecting diagonally placed atoms. (a) Find the longest wavelength electrons that can produce a first-order maximum. (b) If 300eV electrons are used, at what angle from the crystal normal must they be incident to produce a first-order maximum?

<解>：(a) 1.287Å

(b) 11.6^{0}

3-15、What is the wavelength of a hydrogen atom moving with velocity corresponding to the mean kinetic energy for thermal equilibrium at 20℃?

<解>：1.596 Å

3-16、The principal planar spacing in a potassium chloride crystal is 3.14Å. Compare the angle for first-order Bragg reflection from these planes of electrons of kinetic energy 40keV to that of 40keV photons.

<解>：

3-17、Electrons incident on a crystal suffer refraction due to an attractive potential of about 15V that crystals present to electrons (due to the ions in the crystal lattice). If the angle of incidence of an electron beam is 45^{0} and the electrons have an incident energy of 100eV, what is the angle of refraction?

<解>：

3-18、What accelerating voltage would be required for electrons in an electron microscope to obtain the same ultimate resolving power as that which could be obtained from a "γ-ray microscope" using 0.2MeV γ rays?

<解>：37.7kV

3-19、The highest achievable resolving power of a microscope is limited only by the wavelength used; that is, the smallest detail that can be separated is about equal to the wavelength. Suppose we wish to "see" inside an atom. Assuming the atom to have a diameter of 1.0Å, this means that we wish to resolve detail of separation about 0.1Å. (a) If an electron microscope is used, what minimum energy of electrons is needed? (b) If a photon microscope is used, what energy of photons is needed? In what region of the electromagnetic sepectrum are these photons? (c) Which microscope seems more partical for this purpose? Explain.

<解>：(a) $p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\,J-s}{(10^{-11}\,m)c}\,\dfrac{(2.988\times10^{8}\,m/s)}{(1.602\times10^{-13}\,J/MeV)} = \dfrac{0.12400MeV}{c}$

$$E^2 = p^2c^2 + E_0^2$$

$$E^2 = (0.1240)^2 + (0.511)^2 \;\Rightarrow\; E = 0.5258MeV$$

$K = E - E_0 = 0.5258 - 0.5110 = 0.0148MeV = 14.8keV$ ……##

(b) $p = \dfrac{0.12400MeV}{c} = \dfrac{E_{ph}}{c} \;\Rightarrow\; E_{ph} = 124keV$

There are gamma-rays, or hard x-rays…….##

(c) The electron microscope is preferable : the gamma-rays are difficult to focus, and shielding would be required…….##

3-20、Show that for a free particle the uncertainty relation can also be written as $\Delta\lambda\Delta x \ge \dfrac{\lambda^2}{4\pi}$ where Δx is the uncertainty in location of the wave and $\Delta\lambda$ the simulataneous uncertainty in wavelength.

<解>：

3-21、If $\dfrac{\Delta\lambda}{\lambda} = 10^{-7}$ for a photon, what is the simulataneous value of Δx for (a) $\lambda = 5.00\times10^{-4}$ Å (γ ray)? (b) $\lambda = 5.00$ Å (x ray)? (c) $\lambda = 5000$ Å (light)?

<解>：

3-22、In a repetition of Thomson's experiment for measuring e/m for the electron, a beam of $10^4 eV$ electrons is collimated by passage through a slit of which 0.50mm. Why is the beamlike character of the emergent electrons not destroyed by diffraction of the electron wave at this slit?

<解>：

3-23、A 1MeV electron leaves a track in a cloud chamber. The track is a series of water droplets each about $10^{-5}m$ in diameter. Show, from the ratio of the uncertainty in transverse momentum to the momentum of the electron, that the electron path should not noticeably differ from a straight line.

<解>：

3-24、Show that if the uncertainty in the location of a particle is about equal to its de Broglie wavelength, then the uncertainty in its velocity is about equal to one tenth its velocity.

<解>：

3-25、(a) Show that the smallest possible uncertainty in the position of an electron whose speed is given by $\beta = \frac{v}{c}$ is $\Delta x_{\min} = \frac{h}{4\pi m_0 c}(1-\beta^2)^{1/2} = \frac{\lambda_C}{4\pi}\sqrt{1-\beta^2}$ where λ_C is the Compton wavelength $\frac{h}{m_0 c}$. (b) What is the meaning of this equation for $\beta = 0$? For $\beta = 1$?

<解>：

3-26、A microscope using photons is employed to locate an electron in an atom to within a distance of 2.0Å. What is the uncertainty in the velocity of the electron located in this way?

<解>：

3-27、The velocity of a positron is measured to be : $v_x = (4.00 \pm 0.18)\times 10^5 m/\sec$, $v_y = (0.34 \pm 0.12)\times 10^5 m/\sec$, $v_z = (1.41 \pm 0.08)\times 10^5 m/\sec$. Within what minimum volume was the positron located at the moment the measurement was carried out?

<解>：$1.40\times10^{4}\ \overset{0}{A}{}^{3}$

3-28、(a) Consider an electron whose position is somewhere in an atom of diameter 1Å. What is the uncertainty in the electron's momentum? Is this consistent with the binding energy of electrons in atoms? (b) Imagine an electron be somewhere in a nucleus of diameter $10^{-12}cm$. What is the uncertainty in the electron's momentum? Is this consistent with the binding energy of nucleus constituents? (c) Consider now a neutron, or a proton, to be in such a nucleus. What is the uncertainty in the neutron's or proton's, momentum? Is this consistent with the binding energy of nucleus constituents?

<解>：(a) Set $\Delta x = 10^{-10}m$

$$p = \Delta p = \frac{h}{4\pi\Delta x} = \frac{6.626\times10^{-34}J-s}{4\pi(10^{-10}m)}$$

$$p = \frac{5.2728\times10^{-25}kg-m/s}{c}\times\frac{2.988\times10^{8}m/s}{1.602\times10^{-16}J/keV} = \frac{0.9835keV}{c}$$

$$E = (p^2c^2+E_0^2)^{1/2} = [(0.9835)^2+(511)^2]^{1/2} = 511.00095keV$$

$$K = E - E_0 = 511.00095keV - 511keV = 0.95eV$$

Atomic binding energies are on the order of a few electron volts so that this result is consistent with finding electrons inside atoms.

(b) $\Delta x = 10^{-14}m$; hence, $p = 9.835MeV/c$, from (a).

$$E = (p^2c^2+E_0^2)^{1/2} = [(9.835)^2+(0.511)^2]^{1/2} = 9.8812keV$$

$$K = E - E_0 = 9.8812MeV - 0.511MeV = 9.37MeV$$

This is approximately the average binding energy per nucleon, so electrons will tend to escape from nuclei.

(c) For a neutron or proton, $p = 9.835MeV/c$, from (b). Using $938MeV$ as a rest energy,

$$E = (p^2c^2+E_0^2)^{1/2} = [(9.835)^2+(938)^2]^{1/2} = 938.052MeV$$

$$K = E - E_0 = 938.052MeV - 938MeV = 0.052MeV$$

This last result is much less than the average binding energy per nucleon; thus the uncertainty principle is consistent with finding these particles confined inside nuclei.......##

<註>：課本解答 Appendix S，S-1 爲(28a) $0.987 keV/c$, yes (28b) $9.87 MeV/c$, no (28c) $9.87 MeV/c$, yes

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3-29、The lifetime of an excited state of a nucleus is usually about 10^{-12} sec. What is the uncertainty in energy of the γ-ray photon emitted?

<解>：

3-30、An atom in an excited state has a lifetime of 1.2×10^{-8} sec; in a second excited state the lifetime is 2.3×10^{-8} sec. What is the uncertainty in energy for the photon emitted when an electron makes a transition between these two levels?

<解>：$4.17\times10^{-8} eV$

3-31、Use relativistic expressions for total energy and momentum to verify that the group velocity *g* of a matter wave equals the velocity *v* of the associated particle.

<解>：

3-32、The energy of a linear harmonic oscillator is $E=\frac{p_x^2}{2m}+\frac{Cx^2}{2}$. (a) Show, using the uncertainty relation, that can be written as $E=\frac{h^2}{32\pi^2mx^2}+\frac{Cx^2}{2}$. (b) Then show the minimum energy of the oscillator is $\frac{h\nu}{2}$ where $\nu=\frac{1}{2\pi}\sqrt{\frac{C}{m}}$ is the oscillatory frequency. (Hint : This result depends on the $\Delta x\Delta p_x$ product achieving its limiting value $\frac{\hbar}{2}$. Find E in terms of Δx or Δp_x as in part (a), then minimize E with respect to Δx or Δp_x in part (b). Note that classically the minimum energy would be zero.)

<解>：(a) Since $p_x\ge\Delta p_x$ and $x\ge\Delta x$, for the smallest E use $p_x=\Delta p_x$ and $x=\Delta x$

to obtain

$$E=\frac{1}{2m}(\Delta p_x)^2+\frac{1}{2}C(\Delta x)^2$$

With $\Delta p_x \Delta x=\frac{1}{2}\hbar=\frac{h}{4\pi}$

The minimum energy becomes

$$E=\frac{1}{2m}(\frac{h}{4\pi\Delta x})^2+\frac{1}{2}C(\Delta x)^2=\frac{h^2}{32\pi^2 m(\Delta x)^2}+\frac{1}{2}C(\Delta x)^2$$

(b) Set the derivative equal to zero:

$$\frac{dE}{d(\Delta x)}=-\frac{h^2}{16\pi^2 m}\frac{1}{(\Delta x)^3}+C(\Delta x)=0 \;\Rightarrow\; (\Delta x)^2=\frac{h}{4\pi\sqrt{Cm}}$$

Substituting this into the expression for E above gives

$$E_{\min}=\frac{1}{2}h[\frac{1}{2\pi}(\frac{C}{m})^{1/2}]=\frac{1}{2}h\nu \text{ ......##}$$

3-33、A TV tube manufactured is attempting to improve the picture resolution, while keeping costs down, by designing an electron gun that produces an electron beam which will make the smallest possible spot on the face of the tube, using only an electron emitting cathode followed by a system of two well-spaced apertures. (a) Show that there is an optimum diameter for the second aperture. (b) Using reasonable TV tube parameter, estimate the minimum possible spot size.

<解>：

3-34、A boy on top of a ladder of height H is dropping marbles of mass m to the floor and trying to hit a crack in the floor. To aim, he is using equipment of the highest possible precision. (a) Show that the marbles will miss the crack by an average distance of the order of $(\frac{\hbar}{m})^{1/2}(\frac{H}{g})^{1/4}$, where g is the acceleration due to gravity. (b) Using reasonable values of H and m, evaluate this distance.

<解>：

3-35、Show that in order to be able to determine through which slit of a double-slit system each photon passes without destroying the double-slit diffraction pattern, the condition $\Delta y \Delta p_y \le \frac{\hbar}{2}$ must be satisfied. Since this condition violates the uncertainty principle, it cannot be met.

<解>：
