Quantum Physics（量子物理）習題
Robert Eisberg（Second edition）
CH 04：Bohr's model of the atom

4-01、Show, for a Thomson atom, that an electron moving in a stable circular orbit rotates with the same frequency at which it would oscillate in an oscillation through the center along a diameter.
（在湯姆遜原子中，試證明：電子在一穩定圓周軌道上旋轉的頻率與它沿直徑穿過圓心而振動的頻率相同。）

<解>：

4-02、What radius must the Thomson model of a one-electron atom have if it is to radiate a spectral line of wavelength $\lambda = 6000$ Å? Comment on your results.
（在單電子原子的湯姆遜模型中，若它釋放的光譜系波長爲 $\lambda = 6000$ Å，則其半徑爲多少？評論你得到的結果。）

<解>：

4-03、Assume that density of positive charge in any Thomson atom is the same as for the hydrogen atom. Find the radius *R* of a Thomason atom of atomic number *Z* in terms of the radius R_H of the hydrogen atom.

<解>：$Z^{1/3}R_H$

4-04、(a) An α particle of initial velocity *v* collides with a free electron at rest. Show that, assuming the mass of the α particle to be about 7400 electronic masses, the maximum deflection of the α particle is about $10^{-4}\,rad$. (b) Show that the maximum deflection of an α particle that interacts with the positive charge of a Thomson atom of radius 1.0 Å is also about $10^{-4}\,rad$. Hence, argue that $\theta \le 10^{-4}\,rad$ for the scattering of an α particle by a Thomson atom.

<解>：

4-05、Derive (4-5) relating the distance of closest approach and the impact parameter to the scattering angle.

（導出 4-5 式，求出最接近的距離，撞擊參數與散射角間的關係。）

<解>：

4-06、A 5.30MeV α particle is scattered through 60^0 in passing through a thin gold foil. Calculate (a) the distance of closest approach, D, for a head-on collision and (b) the impact parameter, b, corresponding to the 60^0 scattering.

<解>：(a) $4.29\times10^{-14}m$

(b) $3.72\times10^{-14}m$

4-07、What is the distance of closest approach of a 5.30MeV α particle to a copper nucleus in a head-on collision?

（正面撞擊時，5.30MeV 的 α 質點與銅原子核最接近的距離爲何？）

<解>：$1.58\times10^{-14}m$

4-08、Show that the number of α particles scattered by an angle Θ or greater in Rutherford scattering is $(\frac{1}{4\pi\varepsilon_0})^2\pi I\rho t(\frac{zZe^2}{Mv^2})^2\cot^2(\frac{\Theta}{2})$.

<解>：

4-09、The fraction of 6.0MeV protons scattered by a thin gold foil, of density $19.3g/cm^3$, from the incident beam into a region where scattering angles exceed 60^0 is equal to 2.0×10^{-5}, Calculate the thickness of the gold foil, using results of the previous problem.

<解>：9000Å

4-10、A beam of α-particles, of kinetic energy 5.30MeV and intensity 10^4 particle/sec, is incident normally on a gold foil of density $19.3g/cm^3$, atomic weight 197, and thickness $1.0\times10^{-5}cm$. An α particles counter of area $1.0cm^2$ is placed at a distance 10 cm from the foil. If Θ is the angle between the incident beam and a line from the center of the foil to the center of the counter, use the Rutherford scattering differential cross section, (4-9), to find the number of counts per hour for $\Theta=10^0$ and for $\Theta=45^0$. The atomic number of gold is 79.

<解>：By equation 4-8, 4-9

$$dN=(\frac{1}{4\pi\varepsilon_0})^2(\frac{zZe^2}{2Mv^2})^2 In\frac{1}{\sin^4\frac{\Theta}{2}}d\Omega$$

The solid angle of the detector is $d\Omega=\frac{dA}{r^2}=\frac{1.0}{(10)^2}=10^{-2}strad$

Also, $n=(\#nuclei\ \ per\ \ cm^3)(thickness)$

$$n=\frac{19.3}{(197)(1.661\times10^{-24})}(10^{-5})=5.898\times10^{21}m^{-2}$$

Hence, by direct numerical substitution, $dN=6.7920\times10^{-5}\frac{1}{\sin^4\frac{\Theta}{2}}s^{-1}$

The number of counts per hour is $\#=(3600)dN=0.2445\frac{1}{\sin^4\frac{\Theta}{2}}$

This givens : $\Theta=10^0$ $\#=4237$

$\Theta=45^0$ $\#=11.4$……##

<註>：課本解答 Appendix S，S-1 爲 4240, 11.4。

4-11、In the previous problem, a copper foil of density $8.9g/cm^3$, atomic weight 63.6 and thickness $1.0\times10^{-5}cm$ is used instead of gold. When $\Theta=10^0$ we get 820 counts per hour. Find the atomic number of copper.

<解>：

4-12、Prove that Planck's constant has dimensions of angular momentum.
（試證明蒲朗克常數的單位與角動量相同。）

<解>：

4-13、The angular momentum of the electron in a hydrogen-like atom is 7.382×10^{-34} joule-sec. What is the quantum number of the level occupied by the electron?

<解>：$L = n\hbar = \dfrac{nh}{2\pi}$

$$7.382\times10^{-34} = \frac{n}{2\pi}(6.626\times10^{-34})$$

$n = 7$ ……##

4-14、Compare the gravitational attraction of an electron and proton in the ground state of a hydrogen atom to the Coulomb attraction. Are we justified in ignoring the gravitational force?

<解>：$\dfrac{F_{grav}}{F_{coul}} = 4.4\times10^{-40}$, yes

4-15、Show that the frequency of revolution of the electron in the Bohr model hydrogen atom is given by $\nu = \dfrac{2|E|}{hn}$ where E is the total energy of the electron.

<解>：

4-16、Show that for all Bohr orbits the ratio of the magnetic dipole moment of the electronic orbit to its orbital angular momentum has the same value.

<解>：

4-17、(a) Show that in the ground state of the hydrogen atom the speed of the electron can be written as $v = \alpha c$ where α is the fine-structure constant. (b) From the value of α what can you conclude about the neglect of relativistic effects in the

Bohr calculation?

<解>：

4-18、Calculate the speed of the proton in a ground state hydrogen atom.

<解>：The periode of revolution of electron and proton are equal :

$$\frac{2\pi r_e}{v_e}=\frac{2\pi r_p}{v_p} \Rightarrow v_p=(\frac{r_p}{r_e})v_e$$

The motion is about the center of mass of the electron-proton system, so that

$$m_p r_p = m_e r_e \Rightarrow \frac{r_p}{r_e}=\frac{m_e}{m_p}$$

$$\therefore\ v_p=(\frac{m_p}{m_e})v_e=(\frac{m_p}{m_e})(\frac{c}{137})=\frac{1}{1836}\frac{3\times10^8}{137}=1.2\times10^3\,m/s\ \ldots\ldots\#\#$$

4-19、What is the energy, momentum, and wavelength of a photon that is emitted by a hydrogen atom making a direct transition from an excited state with $n=10$ to the ground state? Find the recoil speed of the hydrogen atom in this process.

<解>：13.46eV, 13.46eV/c, 921.2Å, 4.30m/sec

4-20、(a) Using Bohr's formula, calculate the three longest wavelengths in the Balmer series. (b) Between what wavelength limits does the Balmer series lie?

<解>：

4-21、Calculate the shortest wavelength of the Lyman series lines in hydrogen. Of the Paschen series. Of the Pfund series. In what region of the electromagnetic spectrum does each lie?

<解>：

4-22、(a) Using Balmer's generalized formula, show that a hydrogen series identified by the integer *m* of the lowest level occupies a frequency interval range given by $\Delta\nu = \dfrac{cR_H}{(m+1)^2}$. (b) What is the ratio of the range of the Lyman series to that of the Pfund series?

<解>：(a) Frequency of the first line： $\nu_1 = \dfrac{c}{\lambda_1} = cR_H\{\dfrac{1}{m^2} - \dfrac{1}{(m+1)^2}\}$

Frequency of the series limit： $\nu_\infty = \dfrac{c}{\lambda_\infty} = cR_H\{\dfrac{1}{m^2} - 0\}$

Therefore, $\Delta\nu = \nu_\infty - \nu_1 = \dfrac{cR_H}{(m+1)^2}$

(b) $\dfrac{\Delta\nu_{Ly}}{\Delta\nu_{pf}} = \dfrac{\dfrac{cR_H}{(1+1)^2}}{\dfrac{cR_H}{(5+1)^2}} = 9 \ldots\ldots$##

4-23、In the ground state of the hydrogen atom, according to Bohr,s model, what are (a) the quantum number, (b) the orbit radius, (c) the angular momentum, (d) the linear momentum, (e) the angular velocity, (f) the linear speed, (g) the force on the electron, (h) the acceleration of the electron, (i) the kinetic energy, (j) the potential energy, and (k) the total energy? How do the quantities (b) and (k) vary with the quantum number?

<解>：

4-24、How much energy is required to remove an electron from a hydrogen atom in a state with $n = 8$?

<解>：

4-25、A photon ionizes a hydrogen atom from the ground state. The liberated electron recombines with a proton into the first excited state, emitting a 466Å photon. What are (a) the energy of the free electron and (b) the energy of the original photon?

<解>：(a) $E_{ph,2}=\frac{hc}{\lambda_2}=\frac{12400}{466}=26.61eV$

$K=26.61-10.2=16.41eV$

(b) $E_{ph,1}=13.6+16.41=30.01eV$ ……##

<註>：課本解答 Appendix S，S-1 為(a)23.2eV (b)36.8eV

4-26、A hydrogen atom is excited from a state with $n=1$ to one with $n=4$. (a) Calculate the energy that must be absorbed by the atom. (b) Calculate and display on energy-level diagram the different photon energies that may be emitted if the atom returns to $n=1$ state. (c) Calculate the recoil speed of the hydrogen atom, assumed initially at rest, if it makes the transition from $n=4$ to $n=1$ in a single quantum jump.

<解>：

4-27、A hydrogen atom in a state having a binding energy (this is the energy required to remove an electron) of 0.85eV makes a transition to a state with an excitation energy (this is the difference in energy between the state and the ground state) of 10.2eV. (a) Find the energy of the emitted photon. (b) Show this transition on an energy-level diagram for hydrogen, labeling the appropriate quantum numbers.

<解>：

4-28、Show on an energy-level diagram for hydrogen the quantum numbers corresponding to a transition in which the wavelength of the emitted photon is1216Å.

<解>：

4-29、(a) Show that when the recoil kinetic energy of the atom, $\frac{p^2}{2M}$, is taken into account the frequency of a photon emitted in a transition between two atomic levels of energy difference ΔE is reduced by a factor which is approximately $(1-\frac{\Delta E}{2Mc^2})$. (Hint : The recoil momentum is $p=\frac{h\nu}{c}$.) (b) Compare the wavelength of the light emitted from a hydrogen atom in the $3\to 1$ transition when the recoil is taken into account to the wavelength without accounting for recoil.

<解>：

4-30、What is the wavelength of the most energetic photon that can be emitted from a muonic atom with $Z=1$?

<解>：4.90Å

4-31、A hydrogen atom in the ground state absorbs a 20.0eV photon. What is the speed of the liberated electron?

<解>：$1.50\times 10^6 m/\sec$

4-32、Apply Bohr's model to singly ionized helium, that is, to a helium atom with one electron removed. What relationships exist between this spectrum and the hydrogen spectrum?

<解>：

4-33、Using Bohr's model, calculate the energy required to remove the electron from singly ionized helium.

<解>：

4-34、An electron traveling at $1.2\times10^{7} m/\sec$ combines with an alpha particle to from a singly ionized helium atom. If the electron combined directly into the ground level, find the wavelength of the single photon emitted.

<解>：電子的動能爲 $K=(0.511MeV)(\frac{1}{\sqrt{1-\beta^2}}-1)$

$$其中\ \beta=\frac{v}{c}=\frac{1.2\times10^{7}}{2.988\times10^{8}}=0.04$$

$\therefore\ K=409.3eV$

For helium, the second ionization potential from the ground state is

$$E_{ion}=\frac{13.6Z^2}{n^2}=\frac{13.6\times2^2}{1^2}=54.4eV$$

$$E_{ph}=54.4+409.3=463.7eV$$

$$\lambda=\frac{12400}{463.7}=26.74\overset{0}{A}......\#\#$$

4-35、A 3.00eV electron is captured by a bare nucleus of helium. If a 2400Å photon is emitted, into what level was the electron captured?

<解>：$n=5$

4-36、In a Franck-Hertz type of experiment atomic hydrogen is bombarded with electrons, and excitation potentials are found at 10.21V and 12.10V. (a) Explain the observation that three different lines of spectral emission accompany these excitations. (Hint : Draw an energy-level diagram.) (b) Now assume that the energy differences can be expressed as $h\nu$ and find the three allowed values of ν. (c) Assume that ν is the frequency of the emitted radiation and determine the wavelengths of the observed spectral lines.

<解>：

4-37、Assume, in the Franck-Hertz experiment, that the electromagnetic energy emitted by an Hg atom, in giving up the energy absorbed from 4.9eV electrons, equals

$h\nu$, where ν is the frequency corresponding to the 2536Å mercury resonance line. Calculate the value of h according to the Franck-Hertz experiment and compare with Planck's value.

<解>：

4-38、Radiation from a helium ion He^{+} is nearly equal in wavelength to the H_α line (the first line of the Balmer series). (a) Between what states (values of n) does the transition in the helium ion occur? (b) Is the wavelength greater or smaller than of the H_α line? (c) Compute the wavelength difference.

<解>：(a) Hydrogen H_α : $\lambda_H^{-1} = R_H\{\frac{1}{2^2} - \frac{1}{3^2}\}$

Helium, $Z = 2$: $\lambda_{He}^{-1} = 4R_H\{\frac{1}{n_f^2} - \frac{1}{n_i^2}\} = R_H\{\frac{1}{(\frac{n_f}{2})^2} - \frac{1}{(\frac{n_i}{2})^2}\}$

If $\lambda_H = \lambda_{He} \Rightarrow 2 = \frac{n_f}{2} \Rightarrow n_f = 4$

$3 = \frac{n_i}{2} \Rightarrow n_i = 6$

(b) Now take into account the reduced mass μ :

$$R_H = (\frac{1}{4\pi\varepsilon_0})^2 \frac{\mu_H (1)^2 e^4}{4\pi\hbar^3 c} \quad , \quad R_H = (\frac{1}{4\pi\varepsilon_0})^2 \frac{\mu_{He}(2)^2 e^4}{4\pi\hbar^3 c} = \frac{\mu_{He}}{\mu_H}(4R_H)$$

$$\mu_H = \frac{m_e m_p}{m_e + m_p} = m_e(1 - \frac{m_e}{m_p}) \quad , \quad \mu_{He} = \frac{m_e(4m_p)}{m_e + (4m_p)} = m_e(1 - \frac{m_e}{4m_p})$$

$\therefore \mu_{He} > \mu_H$

$\therefore \frac{1}{\lambda_{He}} = R_{He}\{\frac{1}{n_f^2} - \frac{1}{n_i^2}\} > 4R_H\{\frac{1}{n_f^2} - \frac{1}{n_i^2}\}$

Compare to the hydrogen H_α line, the helium 6→4 line wavelength is a little shorter.

$\therefore$ smaller

(c) Since $\lambda \propto \mu^{-1}$ (the factor Z^2 is combined with $\frac{1}{n_f^2} - \frac{1}{n_i^2}$ to give equal values for H and He)

$$\frac{\lambda_H - \lambda_{He}}{\lambda_H} = \frac{\mu_{He} - \mu_H}{\mu_{He}} = 1 - \frac{\mu_H}{\mu_{He}}$$

$$\frac{\Delta\lambda}{\lambda_H} = 1 - \frac{1 - \frac{m_e}{m_p}}{1 - \frac{m_e}{4m_p}} = \frac{3}{4}\frac{m_e}{m_p} = \frac{3}{4}\frac{0.511}{938.3} = 4.084 \times 10^{-4}$$

$$\Delta\lambda = (4.084 \times 10^{-4}) \times (656.3nm) = 0.268nm = 2.68\overset{0}{A} \ldots\ldots\#\#$$

4-39、In stars the Pickering series is found in the He^+ spectrum. It is emitted when the electron in He^+ jumps from higher levels into the level with $n = 4$. (a) Show the exact formula for the wavelength of lines belonging to this series. (b) In what region of the spectrum is the series? (c) Find the wavelength of the series limit. (d) Find the ionization potential, if He^+ is in the ground state, in electron volts.

<解>：(a) $\lambda(\overset{0}{A})\dfrac{3647n^2}{n^2 - 16}$, n=5,6,7,…

(b) visible, infrared

(c) 3647Å

(d) 54.4eV

4-40、Assuming that an amount of hydrogen of mass number three (tritium) sufficient for spectroscopic examination can be put into a tube containing ordinary hydrogen, determine the separation from the normal hydrogen line of the first line of the Balmer series that should be observed. Express the result as a difference in wavelength.

<解>：2.38 Å

4-41、A gas discharge tube contains H^1, H^2, He^3, He^4, Li^6, and Li^7 ions and atoms (the superscript is the atomic mass), with the last four ionized so as to have only one electron. (a) As the potential across the tube is raised from zero, which spectral line should appear first? (b) Given, in order of increasing frequency, the origin of the lines corresponding to the first line of the Lyman series of H^1.

<解>：

4-42、Consider a body rotating freely about a fixed axis. Apply the Wilson-Sommerfield quantization rules, and show that the possible values of the total energy are predicted to be $E=\frac{\hbar^2 n^2}{2I}$ $n=0,1,2,3\ldots$, where I is its rotational inertia, or moment of inertia, about the axis of rotation.

<解>：The momentum associated with the angle θ is $L=I\omega$. The total energy E is $E=K=\frac{1}{2}I\omega^2=\frac{L^2}{2I}$. L is independent of θ for a freely rotating object. Hence, by the Willson-Sommerfeld rule,

$$\oint L d\theta = nh$$

$$L\oint d\theta = L(2\pi) = \sqrt{2IE}\,(2\pi) = nh$$

$$\sqrt{2IE} = n\hbar$$

$$E=\frac{n^2\hbar^2}{2I}\ldots\ldots\#\#$$

4-43、Assume the angular momentum of the earth of mass $6.0\times10^{24}kg$ due to its motion around the sun at radius $1.5\times10^{11}m$ to be quantized according to Bohr's relation $L=\frac{nh}{2\pi}$. What is the value of the quantum number n? Could such quantization be detected?

<解>：