Quantum Physics（量子物理）習題

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CH 08：Magnetic dipole moments, spin and transition rates

8-01、Evaluate the magnetic field produced by a circular current loop at a point on the axis of symmetry far from the loop. Then evaluate the magnetic field produced at the same point by a dipole formed from two separated magnetic monopoles located at the center of the loop and lying along the axis of symmetry. Show that the fields are the same if the current in the loop and its area are related to the magnetic moment of the dipole by (8-2). Can you see how to extend the argument to show that the fields will be the same at all points far from the loop or dipole, and independent of the shape of the loop?

ANS：

8-02、(a) Evaluate the ratio of the orbital magnetic dipole moment to the orbital angular momentum, $\frac{\mu_l}{L}$, for an electron moving in an elliptical orbit of the Bohr-Sommerfeld atom discussed in Section 4-10. (Hint : The area swept out by the radius vector of length *r*, when the angular coordinate increases by the increment $d\theta$, is $dA = r^2 \frac{d\theta}{2}$. Use $L = mr^2 \frac{d\theta}{dt}$ to evaluate $d\theta$ in terms of the time increment dt, and then make the trivial integration.) (b) Compare the results with those of (8-5) for a circular orbit.

ANS：

8-03、The field of an electromagnet is given by $B = 0.02 + 0.0115z^2$, with B in tesla and z = distance in cm from the north pole of the magnet. A magnetic dipole whose moment has magnitude $1.34\times10^{-23} amp-m^2$ is located $8.00cm$ from the north pole, the dipole moment vector at 40^0 to the local magnetic field direction. What are (a) the torque on the dipole, (b) the force on the dipole, and (c) the energy released if the magnetic dipole is turned parallel to the field?

ANS：(a) $6.51\times10^{-24} nt-m$ (b) $1.89\times10^{-22} nt$ (c) $1.48\times10^{-5} eV$

8-04、A beam of hydrogen atoms in their ground state is sent through a Stern-Gerlach magnet, which splits it into two components according to the two spin orientations. One component is stopped by a diaphragm at the end of the magnet, and the other continues into a second Stern-Gerlach magnet which is coaxial with the beam leaving the first magnet, but is rotated relative to the first magnet about

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their approximately common axes through an angle α. There is a second diaphragm fixed on the end of the second magnet which also allows only component to pass. Describe qualitatively how the intensity of the beam passing the second diaphragm depends on α.

ANS：

8-05、Determine the field gradient of a 50 cm long Stern-Gerlach magnet that would produce a 1mm separation at the end of the magnet between the two components of a beam of silver atoms emitted with typical kinetic energy from a $960^{o}C$ oven. The magnetic dipole moment of silver is due to a single $l=0$ electron, just as for hydrogen.

ANS： $29 tesla/m$

8-06、If a hydrogen atom is placed in a magnetic field which is very strong compared to its internal field, its orbital and spin magnetic dipole moments precess independently about the external field, and its energy depends on the quantum number m_l and m_s which specify their components along the external field direction. (a) Evalute the splitting of the energy levels according to the values of m_l and m_s. (b) Draw the pattern of split levels originating from the $n=2$ level, enumerating the quantum numbers of each component of the pattern. (c) Calculate the strength of the external magnetic field that would produce an energy difference between the most widely separated $n=2$ levels which equals the difference between the energies of the $n=1$ and $n=2$ levels in the absence of the field.

ANS：

8-07、Use the procedure of Example 8-3 to estimate the spin-orbit interaction energy in the $n=2$, $l=1$ state of a muonic atom, defined Example 4-9.

ANS： $0.019 eV$

8-08、Prove that the only possible values of the quantum number j from the series $j=l+\frac{1}{2}, l-\frac{1}{2}, l-\frac{3}{2}, \ldots$, that satisfy the inequality $\sqrt{j(j+1)} \geq \left|\sqrt{l(l+1)}-\sqrt{s(s+1)}\right|$ with $s=\frac{1}{2}$, are $j=l+\frac{1}{2}, l-\frac{1}{2}$, if $l \neq 0$, or $j=\frac{1}{2}$, if $l=0$.

ANS：

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8-09、(a) Enumerate the possible values of j and m_j, for the state in which $l=1$, and, of course, $s=\frac{1}{2}$. (b) Draw the corresponding "vector model" figures. (c) Draw a figure illustrating the angular momentum vectors for a typical state. (d) Show also the spin and orbital magnetic dipole moment vectors, and their sum the total magnetic dipole moment vector. (e) Is the total magnetic dipole moment vector antiparallel to the total angular momentum vector?

ANS：

8-10、Consider the states in which $l=4$ and $s=\frac{1}{2}$. For the state with the largest possible j and largest possible m_j, calculate (a) the angle between **L** and **S**, (b) the angle between μ_l and μ_s, and (c) the angle between **J** and the +z axis.

ANS：(a) 74.5^0 (b) 74.5^0 (c) 25.2^0

8-11、Enumerate the possible values of j and m_j for states in which $l=3$ and $s=\frac{1}{2}$.

ANS：

8-12、The relativistic shift in the energy levels of a hydrogen atom due to the relativistic dependence of mass on velocity can be determined by using the atomic eigenfunctions to calculate the expectation value $\overline{\Delta E_{rel}}$ of the quantity $\Delta E_{rel}=E_{rel}-E_{clas}$, the difference between the relativistic and classical expressions for the total energy E. Show that for p not too large $\Delta E_{rel}\simeq-\frac{p^4}{8m^3c^2}=-\frac{E^2+V^2-2EV}{2mc^2}$ so that $\overline{\Delta E_{rel}}\simeq-\frac{E_n^2}{2mc^2}-\frac{e^4}{(4\pi\varepsilon_0)2mc^2}\int\psi^*_{nljm_j}\frac{1}{r}\psi_{nljm_j}d\tau \quad -\frac{E_ne^4}{4\pi\varepsilon_0mc^2}\int\psi^*_{nljm_j}\frac{1}{r}\psi_{nljm_j}d\tau$.

ANS：

8-13、(a) Draw the hydrogen energy-level diagram for all states through $n=2$ as in the right-hand part of Figure 8-11, but with the splitting according to l also shown.

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(b) With arrows connecting pairs of levels, show all the transitions that are allowed by the selection rules.

ANS：

8-14、Verify that the parities of the one-electron atom eigenfunctions ψ_{300}, ψ_{310}, ψ_{320} and ψ_{322} are determined by $(-1)^l$.

ANS：

8-15、(a) Use parity considerations to prove that the first two integrals of the display equation preceding (8-42) both yield zero. (b) Interpret what this means about the existence of atomic electric dipole moments which are static time.

ANS：

8-16、By a straightforward evalution of the electric dipole matrix elements for the eigenfunctions of Table 7-2, show that the selection rule $\Delta l = \pm 1$ of (8-37) is valid for the $n = 2 \to n = 1$ transitions of the hydrogen atom.

ANS：

8-17、Consider the electric dipole moment matrix elements for a charged one-dimensional simple harmonic oscillator making the transitions $n_i = 3$, $n_f = 0$; $n_i = 2$, $n_f = 0$; $n_i = 1$, $n_f = 0$. Use the eigenfunctions of Table 6-1 to show that the matrix elements which are not zero agree with the selection rule $\Delta n = \pm 1$, discussed in Section 4-11. (Hint : Use parity consideration whenever you can.)

ANS：

8-18、(a) Calculate the rate for spontaneous transitions between the $n = 1$ and $n = 0$ states of a simple harmonic oscillator, carrying charge e. Take the mass of the oscillator to be equal to the mass of an atom of some typical ionic molecule, and the restoring force constant C to be $10^3\ joules / m^2$, which is typical for such a molecule. (Hint : Normalized edeigenfunctions must be used.) (b) From the transition rate, estimate the average time required to complete the transition. This is the lifetime of the $n = 1$ vibrational state of the molecule.

ANS：

8-19、Consider enough of the electric dipole moment matrix elements for a charged

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particle in an infinite square well potential, using the eigenfunctions of Section 6-8, to see if there is a selection rule for this system and, if so, to determine what it is.

ANS： $\Delta n = \pm1, \pm3, \pm5, \ldots$

8-20、Find the selection rule for a rigid rotator carrying charge $-e$. Use the eigenfunctions in ϕ found in Problem 23 of Chapter 7. (Note : the selection rule to be found is $\Delta m = \pm1$, not $\Delta m = 0, \pm1$)

ANS：

8-21、Use the result of Problem 8-20 to find the ratio $\frac{R_{12}}{R_{01}}$ of the rates of transition from state 2 to 1 and 1 to 0.

ANS：By Eq. 8-43, $\frac{R_{12}}{R_{01}} = \frac{\nu_{12}^3 P_{12}^2}{\nu_{01}^3 P_{01}^2}$.

But P_{fi} depends only on $\Delta m = m_i - m_f = \pm1$ for both transitions, $P_{12} = P_{01}$.

Hence, $\frac{R_{12}}{R_{01}} = \frac{\nu_{12}^3}{\nu_{01}^3} = (\frac{E_1 - E_2}{E_0 - E_1})^3$, since $\nu = \frac{1}{2\pi}(\frac{\Delta E}{\hbar})$. But $E_m = \frac{m^2\hbar^2}{2I}$,

And therefore $\frac{R_{12}}{R_{01}} = (\frac{1^2 - 2^2}{0^2 - 1^2})^3 = 27$……##

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