Quantum Physics（量子物理）習題

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CH 09：Multielectron atoms – ground states and x-ray excitations

9-01、By going through the procedure indicated in the text, develop the time-independent Schroedinger equation for two noninteracting identical particles in a box, (9-1).

ANS：

9-02、By applying the technique of separation of variables, show that, for a potential of the additive form of (9-2), there are solutions to the two-particle time-independent Schroedinger equation, (9-1), in the product form of (9-3).

ANS：

9-03、Exchange the particle labels in the two probability density functions, obtained from the symmetric and antisymmetric eigenfunctions of (9-8) and (9-9), and show that neither is affected by the exchange.

ANS：The probability densities are

9-04、Verify that expanded from of the three-particle eigenfunction of Example 9-2 is antisymmetric with respect to an exchange of the labels of two particles.

ANS：

9-05、Verify that expanded from of the three-particle eigenfunction of Example 9-2 is identically equal to zero if two particle are in the same space and spin quantum state.

ANS：

9-06、Verify that the $\frac{1}{\sqrt{3!}}$ normalization factor quoted in example 9-2 is correct.

ANS：

9-07、Verify that the expanded from of the three-particle eigenfunction of Example 9-3 is symmetric with respect to an exchange of the labels of two particles.

ANS：

9-08、An α particle contains two protons and two neutrons. Show that if each of its constituents is antisymmetric then it must be symmetric, as stated in Table 9-1. (Hint : Consider a pair of α particles, and the effect of exchanging the labels of all the constituents in one with those of all the constituents in the other.)

ANS：

9-09、Write an expression for the expectation value of the energy associated with the Coulomb interaction between the two electrons of a helium atom in its ground state. Use a space eigenfunction for the system composed of products of one-electron atom eigenfunctions, each of which describes an electron moving independently about the $Z=2$ nucleus. Do not bother to evaluate the expectation value integral, but instead comment on its relation to the energy levels shown in Figure 9-7.

ANS：

9-10、Prove that any two different nondegenerate bound eigenfunctions $\psi_i(x)$ and $\psi_j(x)$ that are solutions to the time-independent Schroedinger equation for the same potential $V(x)$ obey the orthogonality relation $\int_{-\infty}^{\infty}\psi_j^*(x)\psi_i(x)dx=0$ $i\neq j$. (Hint : (i) Write the equations to which $\psi_i(x)$ and $\psi_j(x)$ are solutions, and then take the complex conjugate of the second one to obtain the equation satisfied by ψ_j^*. (ii) Multiply the equation in ψ_i by ψ_j^*, the equation in ψ_j^* by ψ_i, and then subtract. (iii) Integrate, using a relation such as $\psi_j^*\frac{d^2\psi_i}{dx^2}-\psi_i\frac{d^2\psi_j^*}{dx^2}=\frac{d}{dx}(\psi_j^*\frac{d\psi_i}{dx}-\psi_i\frac{d\psi_j^*}{dx})$.) The proof can be extended to include degenerate eigenfunctions, and also unbound eigen –functions that are properly normalized. Can you see how to do this?

ANS：

9-11、(a) By going through the procedure indicated in Section 9-5, develop the time-independent Schroedinger equation for a system of z electrons of an atom moving independently in a set of identical net potentials $V(r)$. (b) Then separate it into a set of Z identical time-indenendent Schroedinger equations, one for each electron. (c) Verify that the form of a typical one is as stated in (9-22). (d)

Compare this form with the time-independent Schroedinger equation for a one-electron atom, (7-12).

ANS：

9-12、(a) Show that there are $N!$ terms in the linear combination for an antisymmetric total eigenfunction describing a system of N independent electrons. (Hint : Consider Example 9-2, and use the mathematical technique of induction.) (b) Evaluate the number of such terms for the case of the argon atom with $Z = 18$. (Hint : Use a mathematical table to evaluate $N!$, or use Stirling's formula, found in most mathematical references, to approximate it.) (c) State briefly the connection between the results of (b) and the procedure used by Hartree to treat the argon atom.

ANS：

9-13、(a) Use information from Figure 9-11 to make a sketch, on semilog paper, of the net potential $V(r)$ for the argon atom. Be sure to determine several values for $\frac{r}{a_0}$ between 0 and 0.25, as this information will be used in Problem 18. (b) Also show the energy levels E_1 and E_2, using estimates from Example 9-5, and energy level E_3, using measured data from Figure 9-15.

ANS：

9-14、(a) Find the value of Z_1 for the helium atom which, when used in the energy equation, (9-27) leads to agreement with the ground state energy shown in Figure 9-6. (b) Compare Z_1 with Z. (c) Is Z_1 meaningful for an atom with as few electrons as helium? Explain briefly.

ANS：2.4

9-15、From Figure 9-6 estimate the average distance between the two electrons in a helium atom (a) in the ground state and (b) in the first excited state. Neglect the exchangeenergy.

ANS：(a) From Fig. 9-6, $E_{coul} = +30eV$

$$E_{coul} = \frac{1}{4\pi\varepsilon_0} = \frac{e^2}{r}$$

$$30 = (9\times10^9)\frac{(1.6\times10^{-19})^2}{r(1.6\times10^{-19})}$$

$$r = 0.048nm$$

(b) $E_{coul} = +9eV \rightarrow r = 0.16nm$ ……##

9-16、(a) Use the Z_n for the argon atom obtained in Example 9-5 in the one-electron atom equation for the radial coordinate expectation value, to estimate the radii of the $n = 1, 2,$ and 3 shells of the atom. (b) Compare the results with Figure 9-10.

ANS：

9-17、Develop a mathematical argument for the tendency, illustrated in Figure 9-12, of an atomic electron with angular momentum L to avoid the point about which it rotates. Treat the electron semiclassically by assuming that it moves around an orbit in a fixed plane passing through the nucleus. (a) Show that its total energy can be written $E = \frac{p_{//}^2}{2m} + [V(r) + \frac{L^2}{2mr^2}] = \frac{p_{//}^2}{2m} + V'(r)$, where $p_{//}$ is its component of linear momentum parallel to its radial coordinate vector of length r. (b) Explain why this indicates that its radial motion is as it would be in a one-dimensional system with potential $V'(r)$. (c) Then show that $V'(r)$ become repulsive for small r because of the dominant behavior of the term $\frac{L^2}{2mr^2}$, sometimes called the centrifugal potential.

ANS：

9-18、(a) Sketch the potentials $V'(r)$ for the argon atom with $l = 0$ and $l = 1$, defined in Problem 17, by adding the corresponding centrifugal potentials to the $V(r)$ obtained in Problem 13. (b) Also sketch the energy level E_2. (b) Show the classical limits of motion, within which $E_2 \geq V'(r)$. (d) Compare these limits with the radial probability densities of Figure 9-10, for $n = 2$, $l = 0$, and $n = 2$, $l = 1$.

ANS：

9-19、Write the configurations for the ground states of ^{28}Ni, ^{29}Cu, ^{30}Zn, ^{31}Ga.

ANS：

9-20、Write the configurations for the ground states of all the lanthanides, making as much use as possible of ditto marks.

ANS：

9-21、Recent work in nuclear physics has led to the prediction that nuclei of atomic number $Z=110$ might be sufficiently stable to allow some of the element $Z=110$ to have survived from the time the elements were created. (a) Predict a likely configuration for this element. (b) Make a prediction of the chemical properties of the element. (c) Where would be a likely place to start searching for traces of it?

ANS：

9-22、(a) From information contained in Figure 9-6 and 9-15, determine the energy required to remove the remaining electron from the ground state of a singly ionized helium atom. (b) Compare this energy predicted by the quantum mechanics of one-electron atoms.

ANS：

9-23、(a) Draw a schematic representation of a standard energy-level diagram for the ^{22}Ti atom, showing the states populated by electrons for a case in which one electron is missing from the K shell. The diagram should be comparable to the one in Figure 9-9 in that it should not attempt to give the energies of the levels to an accurate scale, and no distinction should be made between L_I, L_{II}, and L_{III} levels, etc. (b) Do the same for a case in which one electron is missing from the L shell. (c) Draw a schematic representation of an x-ray energy-level diagram showing the energies of the atom when a hole is in the K or L shell. (d) Compare the utility of the standard and x-ray energy-level diagrams for cases in which a hole is in an inner shell. (e) Also make such a comparison for cases in which a hole is in an outer shell.

ANS：

9-24、The wavelengths of the lines of the K series of ^{74}W are (ignoring fine structure)：for $K_\alpha, \lambda=0.210$ Å; for $K_\beta, \lambda=0.184$ Å; for $K_\gamma, \lambda=0.179$ Å. The wavelength corresponding to the K absorption edge is $\lambda=0.178$ Å. Use this information to construct an x-ray energy-level diagram for ^{74}W.

ANS：

9-25、(a) Make a rough estimate of the minimum accelerating voltage required for an x-ray tube with a ^{26}Fe anode to emit a L_α line of its spectrum. (Hint：As in Example 9-5, $Z_2 \simeq Z-10$.) (b) Also estimate the wavelength of the L_α photon.

ANS：870V

9-26、(a) Use Moseley's data of Figure 9-18 to determine the values of the constants C and a in tis empirical formula, (9-31). (b) Compare these values with those of (9-30), which was derived from the results of the Hartree theory.

ANS：(a) $8.65\times10^{6}m^{-1}$, 1.7

9-27、It is suspected that the cobalt is very poorly mixed with the iron in a block of alloy. To see regions of high cobalt concentration, an x-ray is taken of the block. (a) Predict the energies of the K absorption edges of its constituents. (b) Then determine an x-ray photon energy that would give good contrast. That is, determine an energy of the photon for which the probability of absorption by a cobalt atom would be very different from the probability of absorption by an iron atom.

ANS：(a) $Co:8.50keV$, $Fe:7.83keV$ (b) $8.50keV$

9-28、The Lyman-alpha lifetime in hydrogen is about 10^{-8} sec. From this, find the lifetime for the K_α x-ray transition in lead. (Hint : For the inner electron in lead the wavefunctions are hydrogenic with appropriate effective Z; lifetime $=\frac{1}{R}$, see (8-43).)

ANS：2.44×10^{-16} sec