Quantum Physics（量子物理）習題

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CH 10：Multielectron atoms-optical exciations

10-01、(a) Calculate the wavelength of the $2p$ to $2s$ transition in ^{3}Li. (b) Find the wavelength difference of the two components into which the line is split by the spin-orbit interaction.

ANS：(a) $6700\overset{0}{A}$ (b) $0.152\overset{0}{A}$

10-02、Show that the spin-orbit energy splitting of an alkali atom is given by $\overline{\Delta E} = \frac{\hbar^2}{4m^2c^2}(2l+1)\overline{\frac{1}{r}\frac{dV}{dr}}$ except for $l=0$, in which case the splitting is zero.

ANS：

10-03、(a) Construct an energy-level diagram for ^{11}Na, similar to Figure 10-1, showing all levels lower in energy than the $5s$ level. (b) Devise a way of indicating the spin-orbit splitting of the levels. (Hint：See Figure 10-8.) (c) Indicate which transitions between these levels are allowed by the selection rules.

ANS：

10-04、(a) Predict the values of s', l', j', in the state of maximum energy of two optically active electrons with the quantum numbers $l_1=1$, $s_1=\frac{1}{2}$; $l_2=2$, $s_2=\frac{1}{2}$. (b) Make a sketch, similar to Figure 10-3, which shows the motion of the angular momentum vectors in this state.

ANS：

10-05、Find the possible values of s', l', j' for a configuration with two optically active electrons with quantum number $l_1=1$, $s_1=\frac{1}{2}$; $l_2=3$, $s_2=\frac{1}{2}$. Specify which j' go with each l' and s' combination.

ANS：

10-06、(a) Write down the quantum numbers for the states described in spectroscopic notation as $^{2}S_{3/2}$, $^{3}D_2$, and $^{5}P_3$. (b) Determine if any of these states are impossible, and if so explain why.

ANS：

10-07、Make a sketch, similar to Figure 10-6, which illustrates the LS coupling splittings of the energy levels of a $4s3d$ configuration. Use the Landé interval rule to predict the ratios of the fine-structure splittings of each multiplet, so that they can be drawn to scale. Label the levels with spectroscopic notation.

ANS：

10-08、For an atomic state with quantum numbers $l'=2$, $s'=1$, $j'=3$, find the angle between the total magnetic moment and the direction antiparallel to the total angular momentum. There is no external field present.

ANS：10.0^0

10-09、(a) Use the periodic table of Figure 9-13 to determine the ground state configurations for the atoms ^{12}Mg, ^{13}Al, and ^{14}Si. (b) Then predict the LS coupling quantum numbers for the ground state of each atom. Express you result in spectroscopic notation.

ANS：

10-10、Use the procedure of Example 10-3 to verify the theoretical prediction of Table 10-2 for the Landé interval rule test for the presence of LS coupling in the $4s3d$ configuration of the ^{20}Ca atom.

ANS：

10-11、In an atom which obeys LS coupling, the separations between adjacent energy levels of increasing energy in the five levels of a particular multiplet are in the ratio $1:2:3:4$. Use the procedure of Example 10-4 to assign the quantum numbers s', l', j' to these levels.

ANS：

10-12、Consider a completely filled d subshell, i.e., one containing the ten electrons allowed by the exclusion principle. Ignore the interactions between the electrons, so that the Hartree approximation quantum numbers n, l, m_l, m_s can be used to describe each electron. (a) Show that there is only one possible quantum state for the system that satisfies the exclusion principle. (b) Show that in this state the z components of the total spin angular momentum, the total orbital angular momentum, and the total angular momentum, are all zero. (c) Give an argument showing that these conclusion imply that the magnitudes of the total spin angular momentum, the total orbital angular momentum, and the total angular momentum, are also all zero. (Hint : If an angular momentum vector is not of zero magnitude, but has zero z component in one quantum state, then there are other quantum states in which it has a nonzero z component.) (d) Now consider the interactions between the electrons that are actually present. Can they change the conclusion about the total angular momentum of the subshell? What about the total spin angular momentum and total orbital angular momentum?

ANS：

10-13、(a) Make a rough sketch of the ^6C energy levels in the $2p^2$ and $2p3s$ configurations, using information from Figure 10-8. Indicate the fine-structure splittings of the levels by exaggerating their magnitude. (b) Show all the transitions allowed by the LS coupling selection rules.

ANS：

10-14、(a) Find a state with s', l', j' quantum numbers for which the value of the Landé g factor lies outside the range $g=1$ to $g=2$. (b) Make a sketch, similar to Figure 10-10, which illustrates the angular momentum and magnetic dipole moment vectors for this state.

ANS：

10-15、Consider the $2p3s$ configuration of the ^6C atom, in which the ordering of the energy levels according to s', l', j', and the relative strengths of the dependences of the energy on these quantum numbers, are what is normal for LS coupling. Draw a schematic energy-level diagram for this configuration, like Figure 10-6. Use the same (exaggerated) scale for the fine-structure splitting, given by the Landé interval rule, for all the levels within a given multiplet. (b) Label each level with the spectroscopic notation.

ANS：

10-16、On the energy-level diagram of Problem 15, draw to the same (highly exaggerated) scale the Zeeman dffect splitting, given by the Landé g factor, for each level under the influence of a weak external magnetic field.

ANS：

10-17、(a) Count the total number of components obtained in Problem 16, i.e., the total number of different quantum states in the configuration. (b) Show that this equals the degeneracy of the configuration in the Hartree approximation, i.e., the product of degeneracy factors $2(2l+1)$ for each of the two optically active electrons in the configuration.

ANS：12

10-18、Derive an expression for the Zeeman effect splitting of the levels of a singlet. (Hint : Start at the beginning, and take $s'=0$ so that a simple expression is obtained for the total magnetic dipole moment.)

ANS：

10-19、Give a classical explanation of the normal Zeeman effect based on Faraday's law applied to electrons revolving in circular orbits of constant radius. Show that the correct frequency interval between the three components can be obtained.

ANS：

10-20、(a) Construct a diagram, similar to Figure 10-11, which shows transitions allowed by the selection rules between the singlet states $2p3s\,{}^1P_1$ and $2p^2\,{}^1D_2$ of the ${}^6C$ atom. (b) Verify that the normal Zeeman pattern of three spectral lines will be produced in these transitions. (c) Evaluate the differences in wavelength of these three spectral lines when the atom is in an external field of 0.1 tesla. (Hint : Use a formula for the difference in wavelength derived in Example 10-1). (d) Evaluate the wavelength of the single line obtained when there is no external field, using information from Figure 10-8.

ANS：(c) $1.8\times10^{-3}\overset{0}{A}$ (d) $2000\overset{0}{A}$

10-21、(a) Redraw the energy levels of Figure 10-11, for a case in which the strength of the external magnetic field is increased to the point where the splitting is described by the Paschen-Bach effect. (Hint : Here j' is no longer a useful quantum number.) (b) Redraw the transitions allowed by the m_s' and m_l' selection rules, as in Figure 10-11, and show that they then produce spectral lines which are into only three components.

ANS：

10-22、(a) Use the information contained in Figure 10-8 to estimate the magnitude of the energy associated with the coupling of the two spin angular momenta to form the total spin angular momentum, and with the coupling of the two orbital angular momenta to form the total orbital angular momentum, in $2p^2$ configuration of the ^6C atom. (b) Then estimate the strength of an external field which will produce an energy of orientation with the magnetic dipole moment of each optically active electron larger than the energy estimated in (a). In such a field the couplings of the angular momenta of the optically active electrons are completely destroyed. (c) Is such a field available in the laboratory?

ANS：(a) $1.4eV$ (b) $10^4 tesla$ (c) no