Quantum Physics（量子物理）習題

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CH 13：Solids-conductors and semiconductors

13-01、In Figure 12-23 we illustrate schematically four charge density distributions for valence electrons as functions of the location of atoms, ions, or molecules (shown as dots at the bottom). For each distribution (a), (b), (c), (d), state to which type of binding in solids it most closely corresponds.

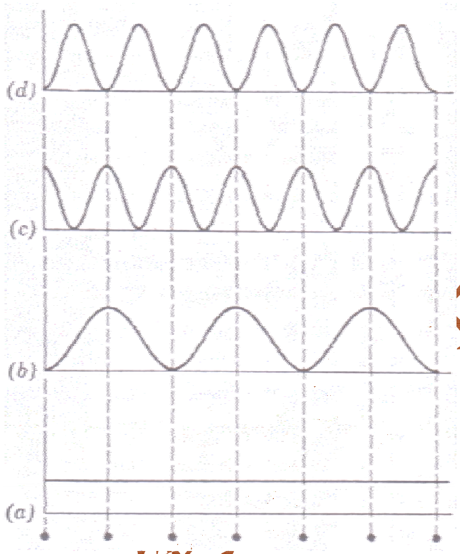

Figure 13-23 Charge densities for valence electrons in four solids considered in Problem 1.

ANS：

13-02、Each element of the row of the periodic table from lithium through neon has a solid form (some at very low temperatures) Solids can also be formed by certain compounds of two elements of this row. For all of these solids, describe the binding and state weather the solid is a metal, a semiconductor, or an insulator.

ANS：

13-03、Describe the binding of solids formed by single elements of the column of the periodic table from carbon through lead, and state whether the solid is a metal, a semiconductor, or an insulator.

ANS：

13-04、Determine the type of bnding in each of the solids described here. (a) Reflects light in the visible; electrical resistivity increase with temperature; melting point below $1000^0 C$. (b) Reflects light in the visible; electrical resistivity descreases with increasing temperature; melting point above 1000^0C. (c) Transmits light in the visible; conducts electricity only at high temperature. (d) Transmits light in the visible; does not conduct electricity at any temperature. (e) Transmits light in the visible; very low melting point.

ANS：(a) metallic (b) covalent(semiconductor) (c) ionic (d) covalent(insulator) (e) molecular

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13-05、The field **E** produced at a point **r** by an electric dipole **p** is given by $\mathbf{E} = -\frac{1}{4\pi\varepsilon_0}(\frac{\mathbf{p}}{r^3} - 3\frac{\mathbf{r}\cdot\mathbf{p}}{r^5}\mathbf{r})$ where the dipole is located at the origin of coordinates. (a) A molecule with an electric dipole moment **p** will induce an electric dipole moment **p'** in a nearby molecule, where $\mathbf{p}' = \alpha\mathbf{E}$, α being the polarizability of the nearby molecule. Show that the mutual potential energy of the interacting dipole is $V = -\mathbf{p}'\cdot\mathbf{E} = -\frac{\alpha}{(4\pi\varepsilon_0)^2}(1+3\cos^2\vartheta)\frac{p^2}{r^6}$ where ϑ is the angle between **r** and **p**. (b) Show that force is attractive and varies as r^{-7}.

ANS：

13-06、Find the order of magnitude of the electric field needed in ionic solids to free electrons from the filled shells of ions. (Hint: Consider the binding energy of an electron and the approximate dimensions of an ion.)

ANS：$10^{10}V/m$

13-07、Find the region of the electromagnetic spectrum at which crystals of Si, Ge, CdS, KCl and Cu become opaque. The band gap energies ε_g are $Si = 1.14eV$; $Ge = 0.67eV$; $CdS = 2.42eV$; $KCl = 7.6eV$; $Cu = 0eV$.

ANS：

13-08、(a) Using classical physics show that the resistivity of a metal near room temperature is proportional to the $\frac{3}{2}$ power of the absolute temperature, in disagreement with the linear temperature dependence experimentally observed. (Hint: Show that $\bar{v} \propto T^{1/2}$ and $\lambda \propto T^{-1}$.) (b) How does the application of the ideas of quantum mechanics and quantum statistics yield the proper temperature dependence of the resistivity?

ANS：

13-09、Compare the values of (a) the drift velocity, (b) the thermal velocity, and (c) the velocity corresponding to the Fermi energy, or Fermi velocity, for electrons in copper at room temperature. (Hint: Use table 11-2. A current of 5amp can easily be carried in a copper wire 0.1cm in diameter.)

ANS：(a) $0.47mm/\sec$ (b) $1.2\times10^5 m/\sec$ (c) $1.6\times10^6 m/\sec$

13-10、Show that, according to the free-electron model, the resistance R of a length L of wire is given by $R=\dfrac{mL}{nAe^2T}$, where A is the cross-sectional area of the wire and T is the mean time between collisions.

ANS：

13-11、An aluminum wire has a resistance of $0.01ohm$, a diameter of $0.83mm$; the mean collision time is $2.0\times10^{-12}\sec$.(a) If the effective electron mass is $0.97m$, find the length of the wire. (b) Find the mean free path for an electron having the Fermi energy. Use data from Table 13-1.

ANS：(a) $65.4m$ (b) $4.4\times\times10^4 \overset{0}{A}$

13-12、Calculate the number of electrons per atom of aluminum that conduct electricity from the value, $-0.3\times10^{-10}m^3/coul$, of the Hall coefficient. The density of aluminum is $2.7\times10^3 kg/m^3$. What does the result suggest about the band structure of aluminum?

ANS：

13-13、(a) Show that the Hall coefficient for a semiconductor in which there is conduction by both holes and electrons is given by $\dfrac{(p\mu_p^2-n\mu_n^2)}{e(p\mu_p+n\mu_n)^2}$. (b) If in a certain semiconductor there is no Hall effect, what fraction of the curried by holes?

ANS：(b) $\sqrt{p}(\sqrt{p}+\sqrt{n})$

13-14、Copper is a monovalent metal with a density of $8g/cm^3$ and an atomic weight of 64. (a) Calculate the Fermi energy in electron volts at 0^0K. (b) Estimate the width of the conduction band.

ANS：

13-15、(a) Calculate the Fermi energy of an alloy of 10% zinc (which is divalent)in copper assuming that the alloy has the same atomic spacing and structure as Cu. (b) How does the width of the conduction band of the alloy compare to that of copper? The assumption used in (a) is not strictly accurate.

ANS：(a) $6.95eV$

13-16、Make an estimate of the width of a conduction band in a metal whose internuclear spacing has the typical value $3.5\times10^{-10}m$.

ANS：

13-17、The Fermi temperature is defined by $T_F=\frac{\varepsilon_F}{k}$. (a) Using Table 11-2, calculate the Fermi temperature for sodium. (b) What does this tell us about the applicability of classical considerations to metals near room temperature? (c) What does this tell us about the density of conduction electrons in a metal at room temperature?

ANS：

13-18、The Fermi energy of lithium is $4.72eV$. (a) Calculate the Fermi velocity. (b) Calculate the de Broglie wavelength of an electron moving at the Fermi velocity and compare it to the interatomic spacing.

ANS：

13-19、The Fermi energy for lithium is $4.72eV$ at $T=0^0K$ .Find the density of states at $3.0eV$.

ANS：$0.756eV^{-1}$

13-20、Calculate an approximate ratio of the electronic specific heat to the lattice specific heat of lithium at room temperature. (Hint: Use the results of Example 13-2, and justify this use.)

ANS：5.5×10^{-3}

13-21、(a) Show that the effect of a lattice periodicity a on periodic potentials having Bloch function solutions is to modulate the free-electron solution so that $\psi(x+a)=\psi(x)e^{ika}$. (b) Show that $e^{ika}=-1$ at the Brillouin zone boundaries. Comment on the meaning of this result.

ANS：

13-22、For a three-dimensional free electron gas confined to a cube, the allowed values of the momentum are distributed uniformly in momentum space. Assume that for each value of the momentum with magnitude less than the Fermi momentum p_F (the momentum corresponding to the Fermi energy) there are two electrons which have that momentum and that there are no electrons with momentum greater than p_F. Show that the number of electrons that have a given x component p_x of momentum is proportional to $1-(\frac{p_x}{p_F})^2$. This result explains the parabolic shape of the angular correlation curves for positron annihilation in metals.

ANS：

13-23、(a) For sodium use the concentration of conduction electrons to estimate the Fermi energy, the Fermi momentum, and the maximum correlation angle ϑ_F for photons from positron annihilation events involving conduction electrons. Sodium has a cubic unit cell with edge $a = 4.22\overset{0}{A}$ and there are two atoms per cube. (b) Repeat the calculations for potassium. Potassium has the same crystalline structure as sodium but the cube edge is $5.22\overset{0}{A}$. (c) In positron annihilation experiments, which of these two metals products the greater fraction of photon pairs with correlation angle greater than ϑ_F?

ANS：

13-24、At what temperature will the number of conduction electrons increase by a factor of 20 over the number at room temperature for germanium? The gap energy is $0.67eV$.

ANS：$377^0 K$

13-25、(a) Show that the number of electrons per unit volume in the conduction band of an intrinsic semiconductor is given by $N_c e^{-\frac{\varepsilon_c-\varepsilon_F}{kT}}$, where $N_c = \frac{2(2\pi mkT)^{3/2}}{h^3}$, and where ε_c is the conduction band-edge energy. (b) Show that the number of holes per unit volume in the valence band of an intrinsic semiconductor is given $N_v e^{-\frac{\varepsilon_F-\varepsilon_v}{kT}}$, where $N_v = \frac{2(2\pi mkT)^{3/2}}{h^3}$, and where ε_v is the valence band-edge energy.

ANS：

13-26、Use the expression for the number of electrons in the conduction band, and the number of holes in the valence band, given in Problem 25, and charge neutrality to find the position of the Fermi energy in an intrinsic semiconductor.

ANS：

13-27、(a) Show that the product of the number of holes in the valence band and the number of electrons in the conduction band depends only on temperature and the gap energy. (b) Show that the conductivity σ of an intrinsic semiconductor can be used to measure the gap energy by calculating $\ln\sigma$.

ANS：

13-28、Write exact expressions for N_d^+ and N_d^0, the concentration of ionized and neutral donors respectively, in a semiconductor doped to a concentration of N_d.

ANS：

13-29、(a) The position of the Fermi energy in a doped semiconductor can be found from the condition of charge neutrality: $N_n + N_a^- = N_p + N_d^+$, where N_n is the number of electrons in the conduction band, N_a^- is the number of ionized acceptors, N_p is the number of holes in the valence band and N_d^+ is the number of ionized donors. Assuming $N_a^- = 0$ and $N_n >> N_p$ show that charge neutrality leads to an equation quadratic in $e^{\frac{\varepsilon_F}{kT}}$ which has the solution $e^{\frac{\varepsilon_F}{kT}} = \dfrac{-1 \pm \sqrt{1 + 4\frac{N_d}{N_c} e^{\frac{\varepsilon_c - \varepsilon_d}{kT}}}}{2e^{-\frac{\varepsilon_d}{kT}}}$ where ε_c is the conduction band-edge energy, and ε_d is the donor-level energy. (b) This equation is soluble in two limits. One is $4\frac{N_d}{N_c} e^{\frac{\varepsilon_c - \varepsilon_d}{kT}} << 1$. This means N_d small or T large. Use a binomial expansion of the square root to show that $N_n = N_d$ and $\varepsilon_F = \varepsilon_c + kT\ln\frac{N_d}{N_c}$. This is the exhaustion region. All the donors are ionized but no electrons are excited from the valence band. (c) In the other limit $4\frac{N_d}{N_c}e^{\frac{\varepsilon_c-\varepsilon_d}{kT}} >> 1$. Also N_d is large and T is small. Show that $N_n = \sqrt{N_d N_c e^{-\frac{\varepsilon_c-\varepsilon_d}{kT}}}$ and $\varepsilon_F = \frac{\varepsilon_c+\varepsilon_d}{2} + kT\ln\frac{N_d}{N_c}$. This is the extrinsic region. Here the donors are being ionized.

ANS：

13-30、Draw an energy-level diagram like that of Figure 12-18 for an n-p-n junction transistor and describe the power amplifier action of the transistor in terms of the figure.

ANS：

13-31、The current which flows in a p-n junction is proportional to the number of electrons in the conduction band. (a) For an unbiased p-n junction, show that the current from the p-region to the n-region is proportional to $e^{-\frac{\varepsilon_g-\varepsilon_F}{kT}}$ and this current is equal to the current from the n-region to the p-region so that no net current flows. (b) When a bias potential V is applied show that the net charge flow per unit area of junction is proportional to $e^{-\frac{\varepsilon_g-\varepsilon_F}{kT}}(e^{\frac{eV}{kT}}-1)$ where eV is positive for forward bias and negative for reverse bias.

ANS：

13-32、A p-n junction is a double layer of opposite charges separated by a small distance and has the properties of a capacitance. The resistivity of a semiconductor can be controlled by doping. Thus the elements in the transistor circuit of Figure 12-24a can be manufactured on a p-n-p semiconductor with appropriate layers etched away as shown in Figure 12-24b. This is an integrated circuit. Label the appropriate parts of Figure 12-24b with the corresponding numbers and letters of Figure 12-24a.

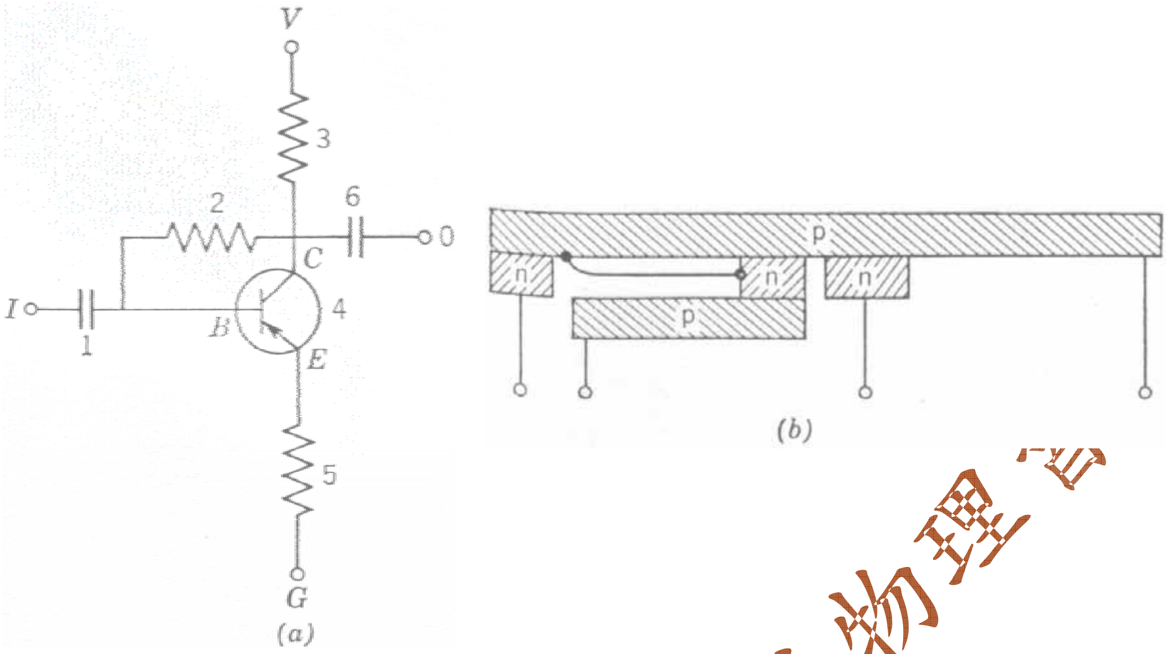

Figure 13-24 An integrated circuit considered in Problem 32.

ANS：

13-33、A tunnel diode junction is approximated by a rectangular barrier 100Å thick and 3.3eV high. If 1.00×10^{25} electrons strike the barrier each second with kinetic energy 3.1eV, and the effective electron mass is 0.30m, what current passes the junction?

ANS：$1.834\times10^{-5}\,amp$
