Quantum Physics（量子物理）習題

Robert Eisberg（Second edition）

CH 14：Solids-superconductors and magnetic properties

14-01、Estimate the size of a Copper pair in mercury by equating the binding energy at $0^0 K$ to the electrostatic repulsion energy between the two electrons.

ANS：$1.3\times10^4 \overset{0}{A}$

14-02、(a) Show, from Maxwell's equations, that resistivity $\rho = 0$ (a perfect conductor) implies that $\mathbf{B} =$ const inside the material. (b) Show, from Maxwell's equations, that $\mathbf{B} = 0$ inside a material (a superconductor) implies that the resistivity of the material is $\rho = 0$.

ANS：

14-03、Show from Lenz's law that the Meissner effect implies perfect conductivity, but that perfect conductivity does not imply the meissner effect.

ANS：

14-04、The critical field of tin at $2^0 K$ is $0.02 weber/m^2$. Drew a graph of the magnetization at $2^0 K$ of a long thin sample of tin as a function of applied field.

ANS：

14-05、Part of the ε versus k diagram for electrons in a superconductor is shown in Figure 14-14. (a) Draw a curve of the density of electrons as a function of ε for a superconductor at $T = 0^0 K$. (b) Draw a graph of the energy necessary to place holes in the superconducting state and electrons in the normal state. This is a graph of $(\varepsilon - \varepsilon_F)$ versus k; ε_F is at the center of the gap for a superconductor. The notion that only electrons are in the normal state and only holes in the superconducting state is not accurate.

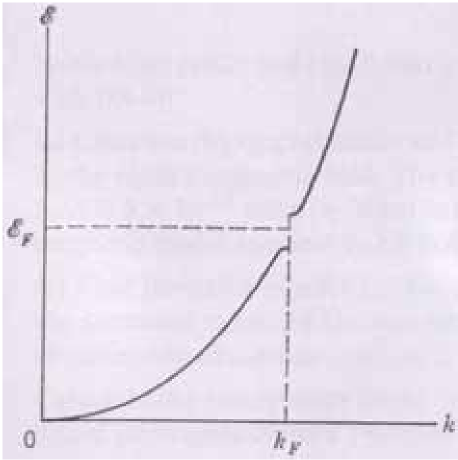

Figure 14-14 The energy as a function of positive wave number for a superconductor; for Problem 5.

ANS：

14-06、When two metals are separated by a very thin insulator, electrons from one metal can tunnel through the insulator to the other metal. Electrons flow until the Fermi levels of the two metals are equal. When a battery is connected between the two metal, as shown in Figure 14-15, the Fermi levels are displaced and a current flows if there are filled electron levels in one metal opposite empty levels in the other metal. Draw current voltage characteristics for the following junctions. (a) Normal metal-normal metal. (b) Normal metal-superconductor. (c) Superconductor-superconductor. (Hint: The Fermi energy of a superconductor lies at the center of the energy gap.)

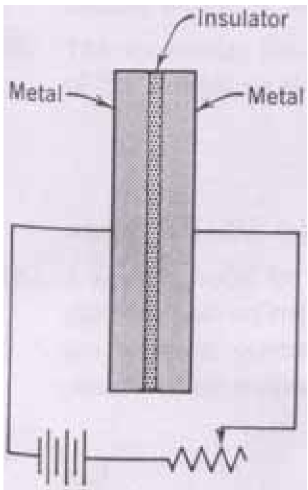

Figure 14-15 Metals separated by a thin insulator; for Problem 6.

ANS：

14-07、Use Faraday's law of induction to show that a hole in a superconductor will trap magnetic flux, i.e., $\frac{dB}{dt}=0$ in the hole. Remember that the electric field $E=0$ in any circuit through the superconductor which encloses the hole, and also that the Meissner effect does not apply to the hole.

ANS：

14-08、Estimate the magnitude of the isotope effect for superconducting materials. Take the critical temperature for naturally occurring vanadium (99.76% V^{51}, with mass $50.9440u$; 0.24% V^{50}, with mass $49.9472u$) to be $5.300^0 K$ precisely. What is the critical temperature for pure V^{50}?

ANS：

14-09、Derive (14-4) for the magnetization, using (14-2) and (14-3).

ANS：

14-10、Show from (14-2) and (14-3) that $\chi=-1$ for a superconductor. Is this result consistent with (14-4)?

ANS：

14-11、(a) Calculate the magnetization of 1 mole of oxygen at standard temperature and pressure in the earth's magnetic field. The susceptibility of oxygen is 2.1×10^{-6} and the earth's field is $5\times10^{-5}\,tesla$. (b) What is the saturation magnetization of 1 mole of oxygen? Its magnetic dipole moment is 2.8 Bohr magnetons.

ANS：(a) $8.4\times10^{-5}\,amp/m$ (b) $700\,amp/m$

14-12、(a) Find the value $\frac{\mu B}{kT}$ for a paramagnetic material with a magnetization one-half the saturated value. (b) Use this result to find the magnetic dipole moment per molecule of postassium chromium sulphate.

ANS：(a) 0.594 (b) $1.43\times10^{-23}\,joule/tesla$

14-13、Calculate the temperature of the sample of Example 14-3 when the magnetic field is reduced isentropically from 1 tesla at $1^0 K$ to 0.01 tesla, assuming Curie's law. (An isentropic process is one in which the populations of the states do not change. Hence the magnetization must remain constant.) This process is called adiabatic demagnetization and is useful in low-temperature physics.

ANS：

14-14、What is the magnetization of the two-level system, discussed in connection with (14-5), when $\mu B >> kT$?

ANS：

14-15、From Figure 14-7 it can be argued that the magnetization due to conduction electrons should be proportional to the number of electrons within μB of the Fermi energy. (a) show that this leads to the susceptibility being given approximately by $\chi = \frac{3N\mu_0\mu_b^2}{2kT_F}$ where N is the number of conduction electrons, μ_0 is the permeability constant, μ_b is the Bohr magneton, and T_F is the Fermi temperature. (b) Evaluate χ for copper.

ANS：

14-16、(a) Show that the specific heat at constant field c_H for the two-level system, discussed in connection with (14-5), given by $c_H = \frac{Nk(\frac{2\mu B}{kT})^2 e^{\frac{2\mu B}{kT}}}{(e^{\frac{2\mu B}{kT}}+1)^2}$ where N is the number of atoms in the system. This is the Schottky specific heat. (Hint: Take the energy of the dipoles aligned parallel to the field to be zero.) (b) What is the temperature dependence of c_H at high and low temperatures? (b) Sketch c_H as a function of T. Estimate (do not calculate) where c_H will be a maximum.

ANS：

14-17、A ferromagnet can be considered to be similar to a paramagnet except that there is an internal molecular field H_W tending to spontaneously align the elementary dipoles. (a) The material will become spontaneously magnetized when the energy of interaction between the dipole and the molecular field is equal to kT_C. Calculate the value of H_W for iron where the magnetic moment is 2.2 Bohr magnetons and T_C is $1000^0 K$. (b) What is the magnetization of a $1cm^3$ sample of iron which has a single domain? (Density$=7.9g/cm^3$;atomic weight$=56$). (c) What is the energy in the field?

ANS：(a) $5.4\times10^8 amp/m$ (b) $1.73\times10^6 amp/m$ (c) $1200\,joule$

14-18、The moleclule field of Problem 17 can be taken as proportional to the magnetization of the sample so that $H_W = \lambda M$. (a) Show that this leads to a susceptibility given by $\chi = \frac{C}{T - T_C}$ where $T_C = C\lambda$. (b) Calculate the value of λ for iron.

ANS：(b) 310

14-19、A simple model for an antiferromagnet is a lattice of two kinds of paramagnetic ions such that the nearest neighbors of A atoms are B atoms. If the antiferromagnetic interactions are between nearest neighbors only，the magnetization of the sample above the Curie point can be written as $TM_A = C'(H - \lambda M_B)$ and $TM_B = C'(H - \lambda M_A)$. Here C' is the Curie constant for one sublattice only. The effective field in sublattice A is $H - \lambda M_B$, and positive λ corresponds to antiferromagnetic interactions between A and B atoms. Show that this leads to a susceptibility above T_C given by $\chi = \frac{C}{T + T_C}$ where $C = 2C'$ and $T_C = C'\lambda$.

ANS：

14-20、Sketch curve of χ^{-1} versus T for $T > T_c$ for (a) a paramagnet, (b) a ferromagnet, and (c) an antiferromagnet, and discuss the meaning of the intercept on the T axis.

ANS：
