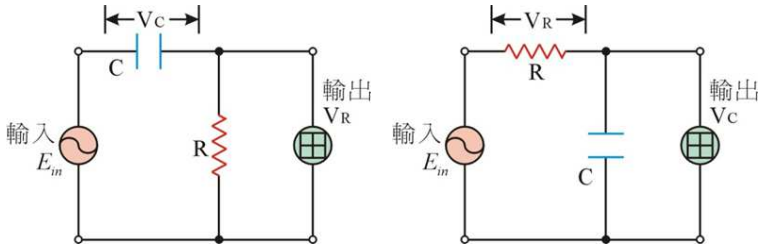
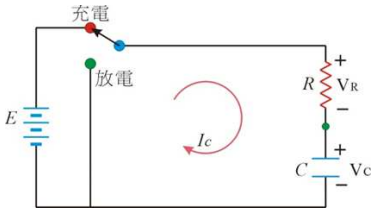
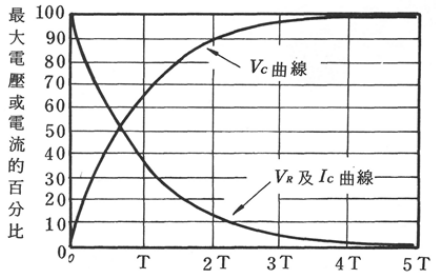
實驗05：
微分器與積分器



RC電路：充電過程





$$E = V_R + V_C = I_C R + \frac{Q}{C}$$

$$I_C = \frac{dQ}{dt}$$

$$E = R\frac{dQ}{dt} + \frac{Q}{C}$$

$$CR\frac{dQ}{dt} + Q - CE = 0$$

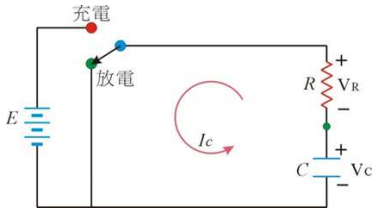
$$Q = CE(1 - e^{\frac{-t}{RC}})$$

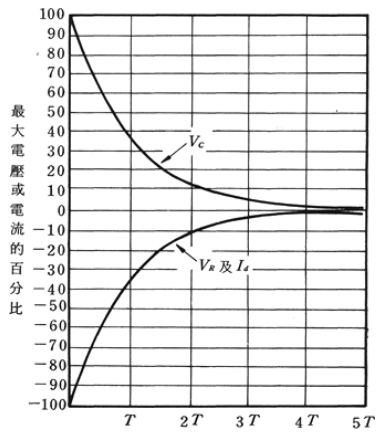
$$I_C = \frac{dQ}{dt} = \frac{E}{R}e^{\frac{-t}{RC}}$$

$$V_R = I_C R = Ee^{\frac{-t}{RC}}$$

$$V_C = E - V_R = E(1 - e^{\frac{-t}{RC}})$$

RC電路：放電過程





當電容充飽後 ➲ $V_C = E$

$$V_R + V_C = I_C R + \frac{Q}{C} = 0$$

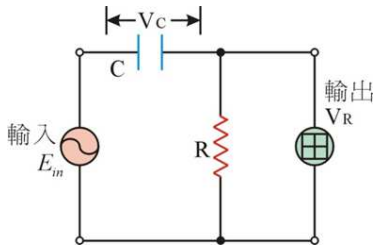
$$Q = CEe^{\frac{-t}{RC}}$$

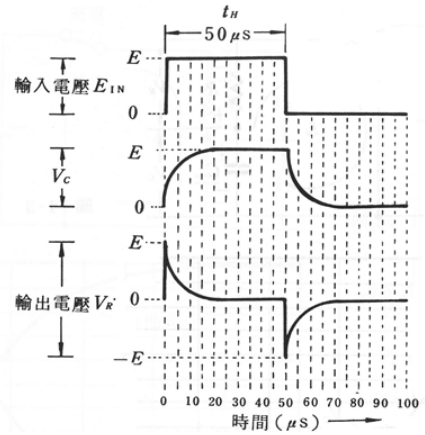
$$I_C = \frac{dQ}{dt} = -\frac{E}{R}e^{\frac{-t}{RC}}$$

$$V_R = I_C R = -Ee^{\frac{-t}{RC}}$$

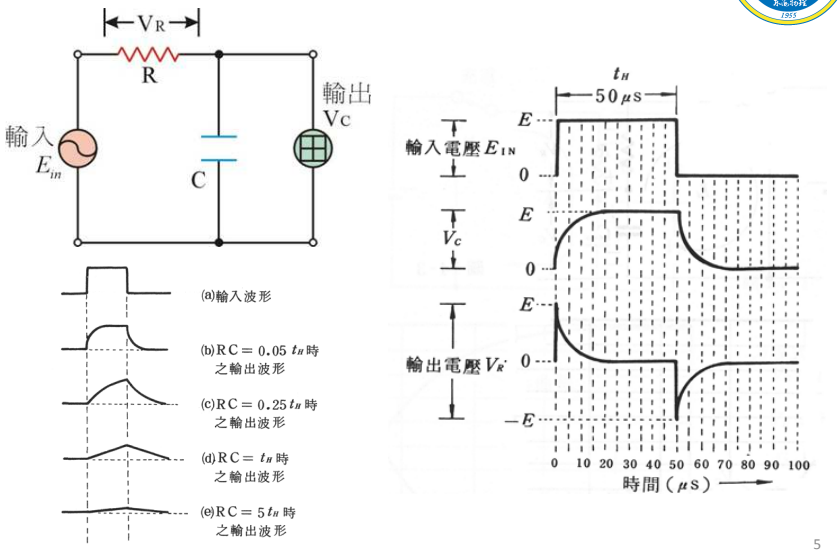
$$V_C = E - V_R = Ee^{\frac{-t}{RC}}$$

微分電路

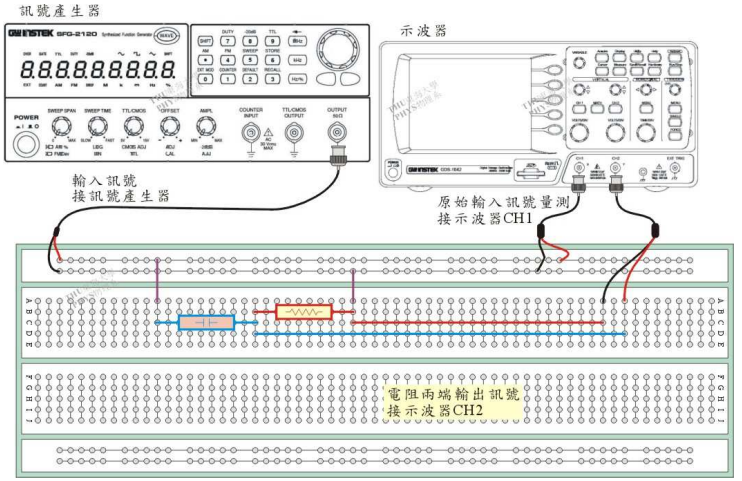




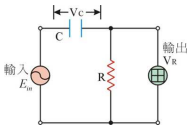
積分電路

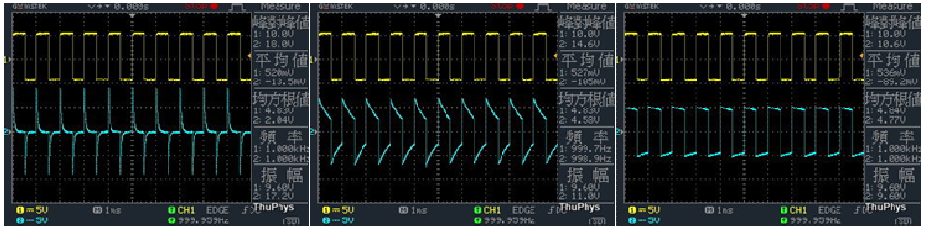


RC電路【【微分電路】】
輸入：方波
輸出：R兩端



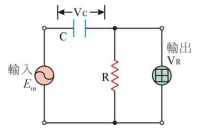
RC電路【【微分電路】】C=0.01uF
輸入：方波
輸出：R兩端

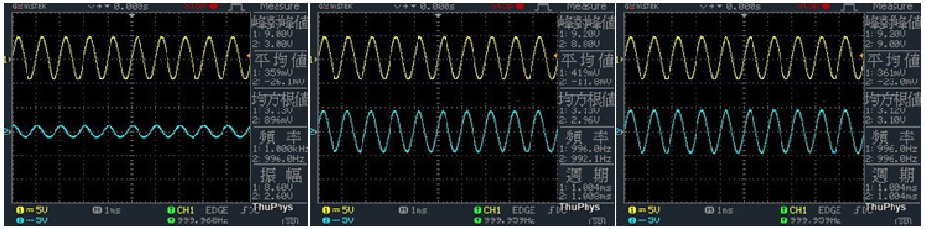




R=5kΩ　R=50kΩ　R=500kΩ

RC電路【【微分電路】】C=0.01uF
輸入：正弦波
輸出：R兩端





R=5kΩ　R=50kΩ　R=500kΩ

RC電路【【微分電路】】C=0.01uF

輸入：正弦波

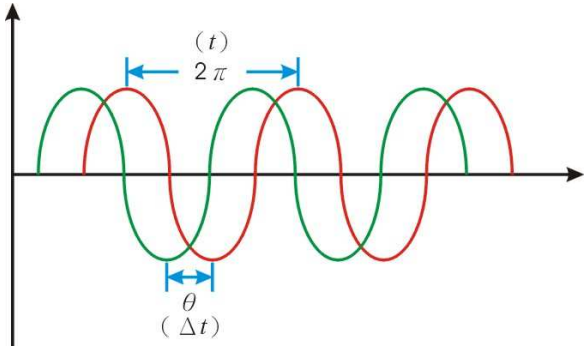
輸出：R兩端

輸入1kHz訊號

➲ t=1ms

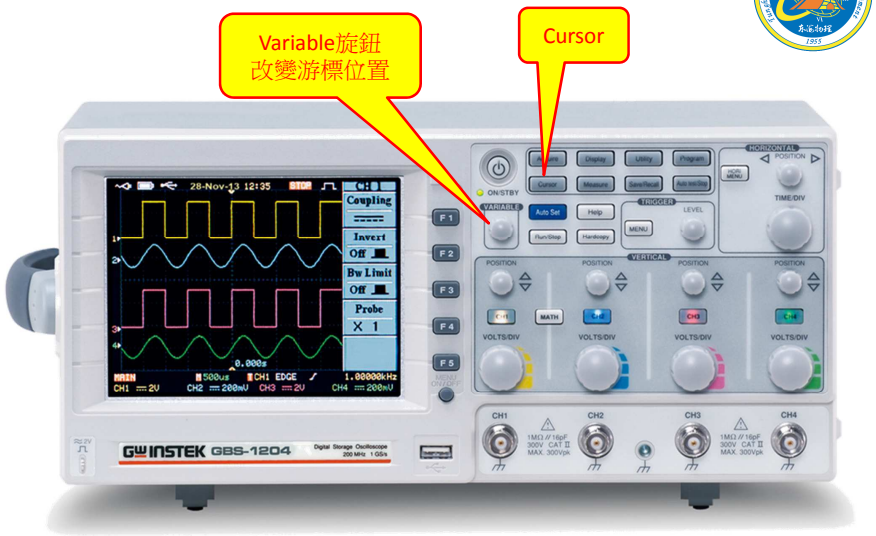
相位差θ：

$$\frac{2\pi}{t} = \frac{\theta}{\Delta t}$$

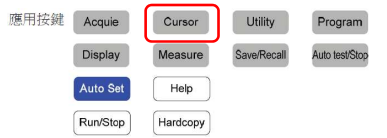


利用示波器Cursor功能，定位 t 與 Δt 位置。

GBS1074示波器 CURSOR游標定位功能

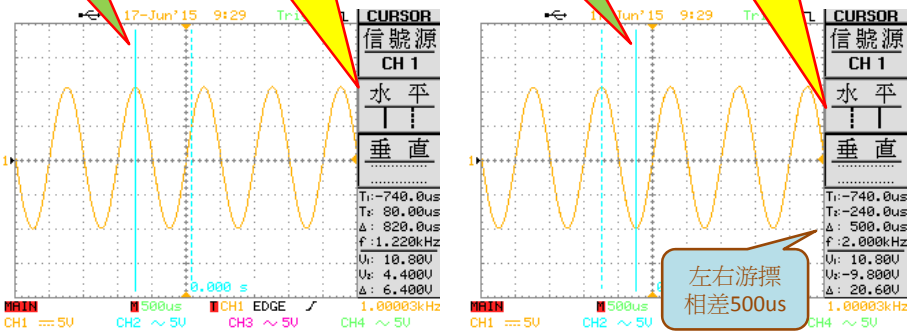


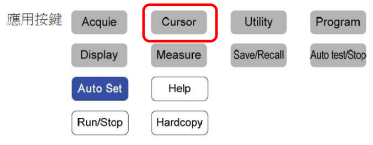
GBS1074示波器 CURSOR游標定位功能



先移動左游標
再移動右游標
記錄兩游標的時間差

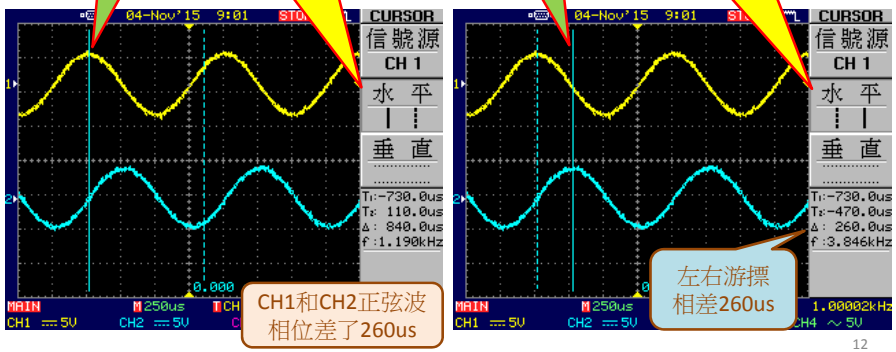
左游標　可移動左游標　右游標　可移動右游標





先移動左游標
再移動右游標
記錄兩游標的時間差

左游標　可移動左游標　右游標　可移動右游標

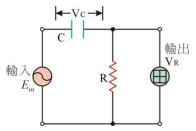


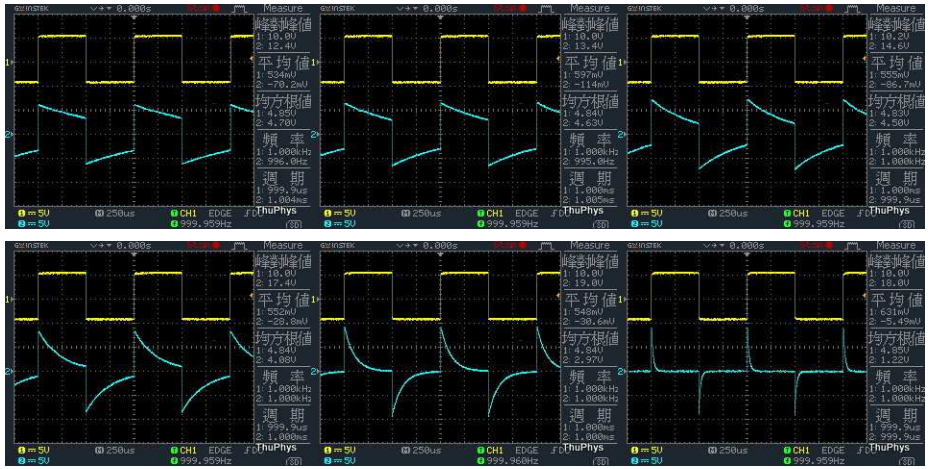
RC電路【【微分電路】】

輸入：方波

輸出：R兩端 R=100Ω，C=0.01uF

改變輸入頻率

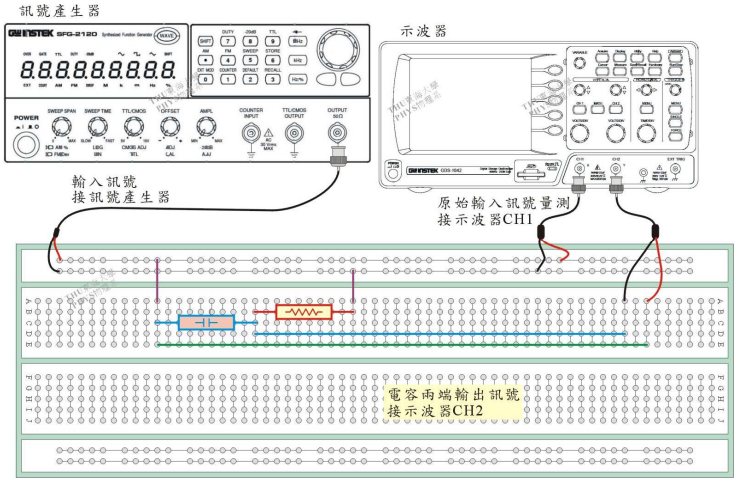




RC電路【【積分電路】】

輸入：方波

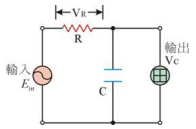
輸出：C兩端

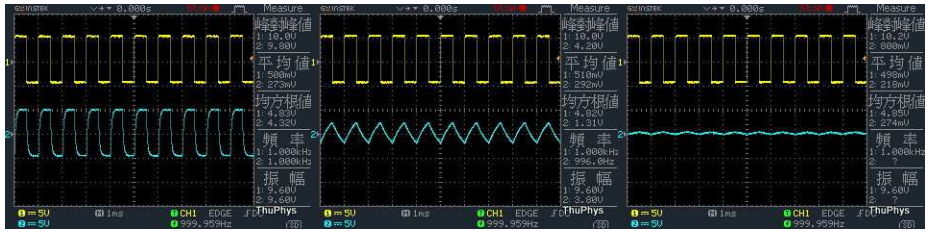


RC電路【【積分電路】】C=0.01uF

輸入：方波

輸出：C兩端



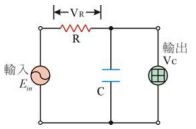


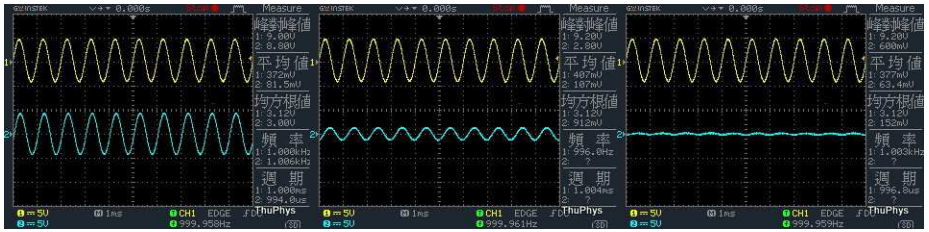
R=5kΩ R=50kΩ R=500kΩ

RC電路【【積分電路】】C=0.01uF

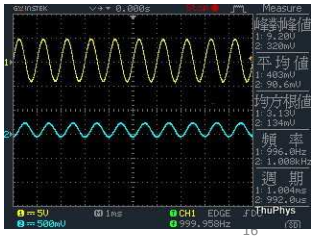
輸入：正弦波

輸出：C兩端



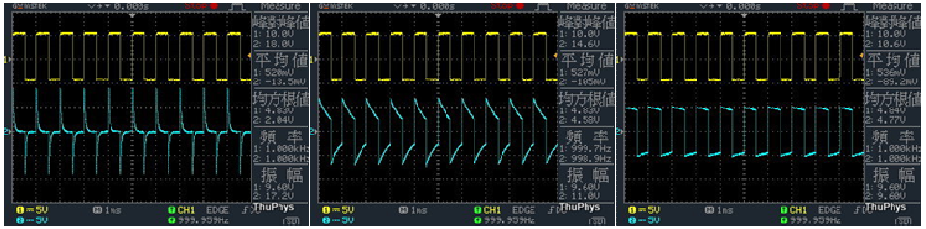


R=5kΩ R=50kΩ R=500kΩ

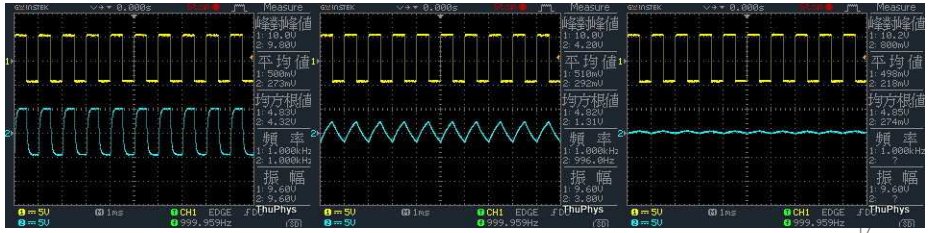


RC電路，C=0.01uF，輸入：方波

輸出：R兩端　微分電路



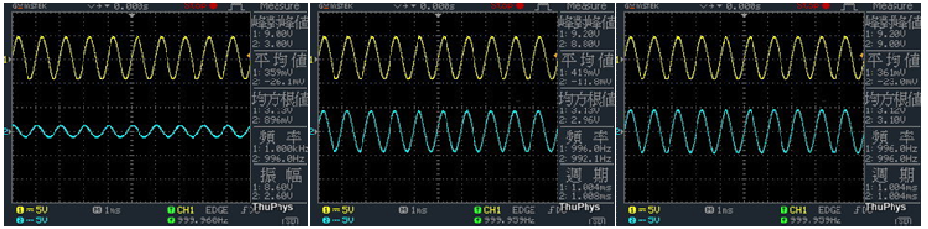
輸出：C兩端　積分電路



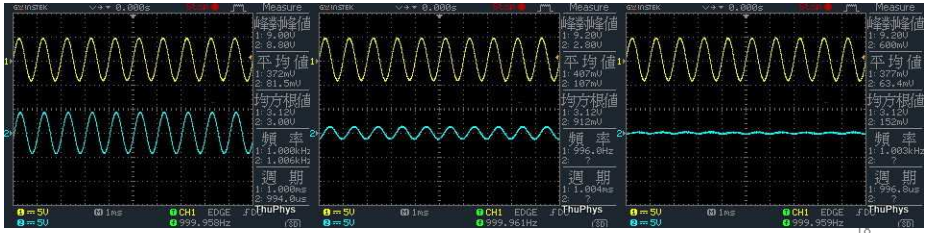
R=5kΩ　R=50kΩ　R=500kΩ

RC電路，C=0.01uF，輸入：正旋波

輸出：R兩端　微分電路



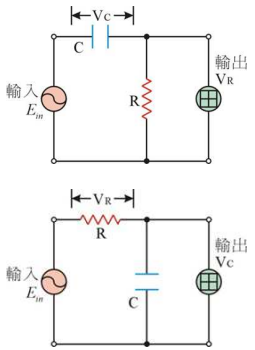
輸出：C兩端　積分電路



R=5kΩ　R=50kΩ　R=500kΩ

RC電路

外加電壓為交流訊號 $V_{AC}=V_0e^{i\omega t}$



$$V_{AC}=V_R+V_C=I_CR+\frac{Q}{C}$$

$$Q=Q_0e^{i\omega t}$$

$$V_0e^{i\omega t}=i\omega RQ_0e^{i\omega t}+\frac{Q_0e^{i\omega t}}{C}$$

$$Q_0=\frac{V_0}{i\omega R+\frac{1}{C}}$$

$$V_C=\frac{Q}{C}=\left(\frac{V_0}{i\omega R+\frac{1}{C}}e^{i\omega t}\right)\frac{1}{C}=\frac{V_{AC}}{i\omega RC+1}$$

電容器上的電壓與外加電壓V_{AC}有相位偏移，偏移量為 $\phi=\tan^{-1}\frac{1}{\omega RC}$

其弦波振幅大小也會衰減為 $|V_C|=\frac{|V_{AC}|}{\sqrt{\omega^2R^2C^2+1}}$

RC電路

外加電壓為交流訊號 $V_{AC}=V_0\cos\omega t$

$$V_{AC}=V_R+V_C=I_CR+\frac{Q}{C}$$

$$R\frac{dQ}{dt}+\frac{Q}{C}=V_0\cos\omega t$$

$$Q=Q_1\cos\omega t+Q_2\sin\omega t$$

$$\frac{dQ}{dt}=-\omega Q_1\sin\omega t+\omega Q_2\cos\omega t$$

$$R\left(-\omega Q_1\sin\omega t+\omega Q_2\cos\omega t\right)+\frac{1}{C}\left(Q_1\cos\omega t+Q_2\sin\omega t\right)=V_0\cos\omega t$$

$$\left(-\omega RQ_1+\frac{Q_2}{C}\right)\sin\omega t+\left(\omega RQ_2+\frac{Q_1}{C}\right)\cos\omega t=V_0\cos\omega t$$

$$-\omega RQ_1+\frac{Q_2}{C}=0 \Rightarrow Q_2=\omega RCQ_1$$

$$\omega RQ_2+\frac{Q_1}{C}=V_0 \Rightarrow \omega^2R^2CQ_1+\frac{Q_1}{C}=V_0$$

$$\omega^2 R^2 C Q_1 + \frac{Q_1}{C} = V_0$$

$$Q_1 = \frac{\frac{V_0}{C}}{\frac{1}{C^2} + \omega^2 R^2} = \frac{CV_0}{1+\omega^2 R^2 C^2}$$

$$Q_2 = \frac{\omega R V_0}{\frac{1}{C^2} + \omega^2 R^2} = \frac{\omega R C^2 V_0}{1+\omega^2 R^2 C^2} = \omega R C Q_1$$

$$Q = Q_1 \cos\omega t + Q_2 \sin\omega t = \frac{CV_0}{1+\omega^2 R^2 C^2}\cos\omega t + \frac{\omega R C^2 V_0}{1+\omega^2 R^2 C^2}\sin\omega t$$

$$= \frac{CV_0}{\sqrt{1+\omega^2 R^2 C^2}}\left(\frac{1}{\sqrt{1+\omega^2 R^2 C^2}}\cos\omega t + \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}\sin\omega t\right)$$

$$= Q_0 \cos(\omega t - \phi)$$

$$Q_0 = \frac{CV_0}{\sqrt{1+\omega^2 R^2 C^2}}$$

$$\phi = \tan^{-1}\frac{1}{\omega RC}$$

$$V_{AC} = V_0 \cos\omega t$$

$$Q = \frac{CV_0}{\sqrt{1+\omega^2 R^2 C^2}}\cos(\omega t - \phi)$$

$$i = \frac{dQ}{dt} = \frac{\omega C V_0}{\sqrt{1+\omega^2 R^2 C^2}}\cos(\omega t - \phi + \tfrac{\pi}{2})$$

$$V_C = \frac{Q}{C} = \frac{V_0}{\sqrt{1+\omega^2 R^2 C^2}}\cos(\omega t - \phi)$$

R（kΩ）	C（uF）	V_C振幅	V_R振幅
5	0.01	0.95	0.30
50	0.01	0.30	0.95
500	0.01	0.03	1.00

$$V_R = iR = \frac{\omega RCV_0}{\sqrt{1+\omega^2 R^2 C^2}}\cos(\omega t - \phi + \tfrac{\pi}{2})$$

電源電壓V_{AC}領先電容器電壓V_C，相位ϕ。
電源電壓V_{AC}領先電阻器電壓V_R，相位ϕ-90度。

$$\phi = \tan^{-1}\frac{1}{\omega RC}$$

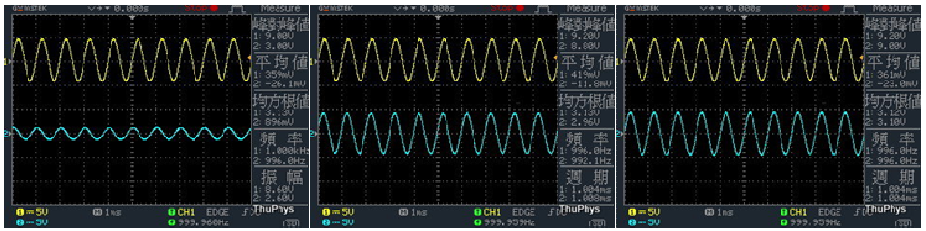
$$\omega = 2\pi f = 2\pi * 1000Hz$$

R（kΩ）	C（uF）	V_C相位差	V_R相位差
5	0.01	72.57	17.43
50	0.01	17.67	72.33
500	0.01	1.82	88.18

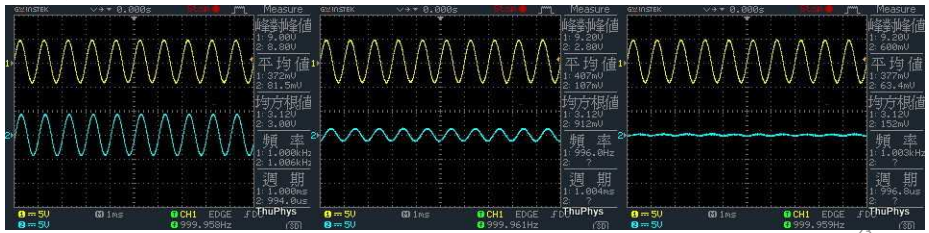
RC電路，C=0.01uF，輸入：正旋波

R（kΩ）	C（uF）	V_C振幅	V_R振幅
5	0.01	0.95	0.30
50	0.01	0.30	0.95
500	0.01	0.03	1.00

輸出：R兩端　微分電路



輸出：C兩端　積分電路



R=5kΩ　R=50kΩ　R=500kΩ

我們沒有最好
只有追求更好

有空繼續補～～

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