



Differentiation rules

General formulas

- 1、 $\frac{d}{dx}(c) = 0$
- 2、 $\frac{d}{dx}[cf(x)] = cf'(x)$
- 3、 $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
- 4、 $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
- 5、 $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$
(Product Rule)
- 6、 $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
(Quotient Rule)
- 7、 $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)
- 8、 $\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)

Exponential and Logarithmic Functions

- 9、 $\frac{d}{dx}(e^x) = e^x$
- 10、 $\frac{d}{dx}(a^x) = a^x \ln a$
- 11、 $\frac{d}{dx} \ln|x| = \frac{1}{x}$
- 12、 $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Trigonometric Functions

- 13、 $\frac{d}{dx}(\sin x) = \cos x$
- 14、 $\frac{d}{dx}(\cos x) = -\sin x$
- 15、 $\frac{d}{dx}(\tan x) = \sec^2 x$
- 16、 $\frac{d}{dx}(\csc x) = -\csc x \cot x$
- 17、 $\frac{d}{dx}(\sec x) = \sec x \tan x$
- 18、 $\frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trigonometric Functions

- 19、 $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- 20、 $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
- 21、 $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- 22、 $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$
- 23、 $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
- 24、 $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

Hyperbolic Functions

- 25、 $\frac{d}{dx}(\sinh x) = \cosh x$
- 26、 $\frac{d}{dx}(\cosh x) = \sinh x$
- 27、 $\frac{d}{dx}(\tanh x) = \text{sech}^2 x$
- 28、 $\frac{d}{dx}(\text{csch } x) = -\text{csch } x \coth x$
- 29、 $\frac{d}{dx}(\text{sech } x) = -\text{sech } x \tanh x$
- 30、 $\frac{d}{dx}(\text{coth } x) = -\text{csch}^2 x$

Inverse Hyperbolic Functions

- 31、 $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$
- 32、 $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$
- 33、 $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$
- 34、 $\frac{d}{dx}(\text{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$
- 35、 $\frac{d}{dx}(\text{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$
- 36、 $\frac{d}{dx}(\text{coth}^{-1} x) = \frac{1}{1-x^2}$



Table of Integrals

Basic Forms

- 1、 $\int u dv = uv - \int v du$
- 11、 $\int \csc u \cot u du = -\csc u + C$
- 2、 $\int u^2 du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
- 12、 $\int \tan u du = \ln|\sec u| + C$
- 3、 $\int \frac{du}{u} = \ln|u| + C$
- 13、 $\int \cot u du = \ln|\sin u| + C$
- 4、 $\int e^u du = e^u + C$
- 14、 $\int \sec u du = \ln|\sec u + \tan u| + C$
- 5、 $\int a^u du = \frac{a^u}{\ln a} + C$
- 15、 $\int \csc u du = \ln|\csc u - \cot u| + C$
- 6、 $\int \sin u du = -\cos u + C$
- 16、 $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1} \frac{u}{a} + C$
- 7、 $\int \cos u du = \sin u + C$
- 17、 $\int \frac{du}{a^2+u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
- 8、 $\int \sec^2 u du = \tan u + C$
- 18、 $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$
- 9、 $\int \csc^2 u du = -\cot u + C$
- 19、 $\int \frac{du}{a^2-u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$
- 10、 $\int \sec u \tan u du = \sec u + C$
- 20、 $\int \frac{du}{u^2-a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$

Forms Involving $\sqrt{a^2+u^2}, a > 0$

- 21、 $\int \sqrt{a^2+u^2} du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2+u^2}) + C$
- 22、 $\int u^2 \sqrt{a^2+u^2} du = \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2+u^2}) + C$

$$23 \cdot \int \frac{\sqrt{a^2+u^2}}{u} du = \sqrt{a^2+u^2} - a \ln \left| \frac{a+\sqrt{a^2+u^2}}{u} \right| + C$$

$$24 \cdot \int \frac{\sqrt{a^2+u^2}}{u^2} du = -\frac{\sqrt{a^2+u^2}}{u} + \ln(u+\sqrt{a^2+u^2}) + C$$

$$25 \cdot \int \frac{du}{\sqrt{a^2+u^2}} = \ln(u+\sqrt{a^2+u^2}) + C$$

$$26 \cdot \int \frac{u^2 du}{\sqrt{a^2+u^2}} = \frac{u}{2} \sqrt{a^2+u^2} - \frac{a^2}{2} \ln(u+\sqrt{a^2+u^2}) + C$$

$$27 \cdot \int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2+u^2}+a}{u} \right| + C$$

$$28 \cdot \int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2 u} + C$$

$$29 \cdot \int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$$

Forms Involving $\sqrt{a^2-u^2}$, $a > 0$

$$30 \cdot \int \sqrt{a^2-u^2} du = \frac{u}{2} \sqrt{a^2-u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$31 \cdot \int u^2 \sqrt{a^2-u^2} du = \frac{u}{8} (2u^2-a^2) \sqrt{a^2-u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$32 \cdot \int \frac{\sqrt{a^2-u^2}}{u} du = \sqrt{a^2-u^2} - a \ln \left| \frac{a+\sqrt{a^2-u^2}}{u} \right| + C$$

$$33 \cdot \int \frac{\sqrt{a^2-u^2}}{u^2} du = -\frac{\sqrt{a^2-u^2}}{u} - \sin^{-1} \frac{u}{a} + C$$

$$34 \cdot \int \frac{u^2 du}{\sqrt{a^2-u^2}} = -\frac{u}{2} \sqrt{a^2-u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$35 \cdot \int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2-u^2}}{u} \right| + C$$

$$36 \cdot \int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2 u} + C$$

$$37 \cdot \int (a^2-u^2)^{3/2} du = -\frac{u}{8} (2u^2-5a^2) \sqrt{a^2-u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$38 \cdot \int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$$

Forms Involving $\sqrt{u^2-a^2}$, $a > 0$

$$39 \cdot \int \sqrt{u^2-a^2} du = \frac{u}{2} \sqrt{u^2-a^2} - \frac{a^2}{2} \ln |u+\sqrt{u^2-a^2}| + C$$

$$40 \cdot \int u^2 \sqrt{u^2-a^2} du = \frac{u}{8} (2u^2-a^2) \sqrt{u^2-a^2} - \frac{a^4}{8} \ln |u+\sqrt{u^2-a^2}| + C$$

$$41 \cdot \int \frac{\sqrt{u^2-a^2}}{u} du = \sqrt{u^2-a^2} - a \cos^{-1} \frac{a}{|u|} + C$$

$$42 \cdot \int \frac{\sqrt{u^2-a^2}}{u^2} du = -\frac{\sqrt{u^2-a^2}}{u} + \ln |u+\sqrt{u^2-a^2}| + C$$

$$43 \cdot \int \frac{du}{\sqrt{u^2-a^2}} = \ln |u+\sqrt{u^2-a^2}| + C$$

$$44 \cdot \int \frac{u^2 du}{\sqrt{u^2-a^2}} = \frac{u}{2} \sqrt{u^2-a^2} + \frac{a^2}{2} \ln |u+\sqrt{u^2-a^2}| + C$$

$$45 \cdot \int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2 u} + C$$

$$46 \cdot \int \frac{du}{(u^2-a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C$$

Forms Involving $a+bu$

$$47 \cdot \int \frac{udu}{a+bu} = \frac{1}{b^2} (a+bu - a \ln |a+bu|) + C$$

$$48 \cdot \int \frac{u^2 du}{a+bu} = \frac{1}{2b^3} [(a+bu)^2 - 4a(a+bu) + 2a^2 \ln |a+bu|] + C$$

$$49 \cdot \int \frac{du}{u(a+bu)} = \frac{1}{a} \ln \left| \frac{u}{a+bu} \right| + C$$

$$50 \cdot \int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a+bu}{u} \right| + C$$

$$51 \cdot \int \frac{udu}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2} \ln |a+bu| + C$$

$$52 \cdot \int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2} \ln \left| \frac{a+bu}{u} \right| + C$$

$$53 \cdot \int \frac{u^2 du}{(a+bu)^2} = \frac{1}{b^3} \left(a+bu - \frac{a^2}{a+bu} - 2a \ln|a+bu| \right) + C$$

$$54 \cdot \int u\sqrt{a+bu} du = \frac{2}{15b^2} (3bu-2a)(a+bu)^{3/2} + C$$

$$55 \cdot \int \frac{udu}{\sqrt{a+bu}} = \frac{2}{3b^2} (bu-2a)\sqrt{a+bu} + C$$

$$56 \cdot \int \frac{u^2 du}{\sqrt{a+bu}} = \frac{2}{15b^3} (8a^2+3b^2u^2-4abu)\sqrt{a+bu} + C$$

$$57 \cdot \int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}} \right| + C, \text{ if } a > 0$$

$$= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bu}{-a}} + C, \text{ if } a < 0$$

$$58 \cdot \int \frac{\sqrt{a+bu}}{u} du = 2\sqrt{a+bu} + a \int \frac{du}{u\sqrt{a+bu}}$$

$$59 \cdot \int \frac{\sqrt{a+bu}}{u^2} du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2} \int \frac{du}{u\sqrt{a+bu}}$$

$$60 \cdot \int u^n \sqrt{a+bu} du = \frac{2}{b(2n+3)} [u^n(a+bu)^{3/2} - na \int u^{n-1} \sqrt{a+bu} du]$$

$$61 \cdot \int \frac{u^n du}{\sqrt{a+bu}} = \frac{2u^n \sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)} \int \frac{u^{n-1} du}{\sqrt{a+bu}}$$

$$62 \cdot \int \frac{du}{u^n \sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a+bu}}$$

Trigonometric Forms

$$63 \cdot \int \sin^2 u du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$$

$$64 \cdot \int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$$65 \cdot \int \tan^2 u du = \tan u - u + C$$

$$66 \cdot \int \cot^2 u du = -\cot u - u + C$$

$$67 \cdot \int \sin^3 u du = -\frac{1}{3}(2+\sin^2 u)\cos u + C$$

$$68 \cdot \int \cos^3 u du = \frac{1}{3}(2+\cos^2 u)\sin u + C$$

$$69 \cdot \int \tan^3 u du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$$

$$70 \cdot \int \cot^3 u du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$$

$$71 \cdot \int \sec^3 u du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$$

$$72 \cdot \int \csc^3 u du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$$

$$73 \cdot \int \sin^n u du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u du$$

$$74 \cdot \int \cos^n u du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u du$$

$$75 \cdot \int \tan^n u du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u du$$

$$76 \cdot \int \cot^n u du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u du$$

$$77 \cdot \int \sec^n u du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u du$$

$$78 \cdot \int \csc^n u du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u du$$

$$79 \cdot \int \sin au \sin bu du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$$

$$80 \cdot \int \cos au \cos bu du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$$

$$81 \cdot \int \sin au \cos bu du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$82 \cdot \int u \sin u du = \sin u - u \cos u + C$$

$$83 \cdot \int u \cos u du = \cos u + u \sin u + C$$

$$84 \cdot \int u^n \sin u du = -u^n \cos u + n \int u^{n-1} \cos u du$$

$$85 \cdot \int u^n \cos u du = u^n \sin u - n \int u^{n-1} \sin u du$$

$$86 \cdot \int \sin^n u \cos^m u du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u du$$

$$= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u du$$

Inverse Trigonometric Forms

$$87 \cdot \int \sin^{-1} u du = u \sin^{-1} u + \sqrt{1-u^2} + C$$

$$88 \cdot \int \cos^{-1} u du = u \cos^{-1} u - \sqrt{1-u^2} + C$$

$$89 \cdot \int \tan^{-1} u du = u \tan^{-1} u - \frac{1}{2} \ln(1+u^2) + C$$

$$90 \cdot \int u \sin^{-1} u du = \frac{2u^2-1}{4} \sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$91 \cdot \int u \cos^{-1} u du = \frac{2u^2-1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$92 \cdot \int u \tan^{-1} u du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$$

$$93 \cdot \int u^n \sin^{-1} u du = \frac{1}{n+1} [u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}}], n \neq -1$$

$$94 \cdot \int u^n \cos^{-1} u du = \frac{1}{n+1} [u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}}], n \neq -1$$

$$95 \cdot \int u^n \tan^{-1} u du = \frac{1}{n+1} [u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} du}{1+u^2}], n \neq -1$$

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Exponential and Logarithmic Forms

96. $\int u e^{au} du = \frac{1}{a^2} (au - 1)e^{au} + C$

97. $\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$

98. $\int e^{au} \sin budu = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$

99. $\int e^{au} \cos budu = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$

100. $\int \ln u du = u \ln u - u + C$

101. $\int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$

102. $\int \frac{1}{u \ln u} du = \ln |\ln u| + C$

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Hyperbolic Forms

103. $\int \sinh u du = \cosh u + C$ 104. $\int \cosh u du = \sinh u + C$

105. $\int \tanh u du = \ln \cosh u + C$ 106. $\int \coth u du = \ln |\sinh u| + C$

107. $\int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$ 108. $\int \operatorname{csch} u du = \ln \left| \tanh \frac{1}{2} u \right| + C$

109. $\int \operatorname{sech}^2 u du = \tanh u + C$ 110. $\int \operatorname{csch}^2 u du = -\coth u + C$

111. $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$ 112. $\int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$

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Forms Involving $\sqrt{2au - u^2}$, $a > 0$

113. $\int \sqrt{2au - u^2} du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$

114. $\int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$

115. $\int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$

116. $\int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1} \left(\frac{a-u}{a} \right) + C$

117. $\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1} \left(\frac{a-u}{a} \right) + C$

118. $\int \frac{udu}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$

119. $\int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u+3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$

120. $\int \frac{du}{u\sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$

